

ANALYTICAL-NUMERICAL METHOD FOR SOLVING NONLINEAR DYNAMICAL SYSTEMS

Usman, Mustapha Adeyemi and Alaba, Adedeji Opeyemi
 Department of Mathematical Science, Olabisi Onabanjo University Ago Iwoye, Ogun State Nigeria.
 E-mail: usman.mustapha@oouagoiwoye.edu.ng

ABSTRACT

This paper investigates the analytical-numerical method for solving nonlinear dynamical systems. The governing partial differential equation of order four was transformed to Ordinary differential equation using analytical method. The finite difference method was used to transform the approximate governing equation. It was shown from the graph of deflection against distance that the deflection increases as the value of distance increases and also from the graph of deflection against time, the deflection increases with increase in time. The result is in agreement with the existing results.

Keyword: Deflection, Dynamical Systems, Finite difference Method, Load, Nonlinear Equations, Systems

Accepted Date:12 September 2017:

INTRODUCTION

The moving load is an unavoidable difficulty in structural dynamics. The dynamic behaviour of beams on elastic foundations subjected to moving loads or masses has been investigated by many researchers in engineering, especially in Railway Engineering. The modern trend towards higher speeds in the railways has further intensified the research in order to accurately predict the vibration behaviour of the railway track. These studies mostly considered the Winkler elastic foundation model that consists of infinite closely-spaced linear springs subjected to a moving load.

The dynamic response of structures carrying moving masses is a problem of wide spread practical significance. A lot of hard work has been done during the last 100 years relating with the dynamic response of railways bridges and highway bridges under the effect of moving loads. Beam type structures are widely used in many branches of civil, mechanical and aerospace engineering. The dynamic effect of moving loads was not known until mid-nineteenth century. When the Stephenson's bridge across river Dee Chester in England in 1947 collapsed, it motivates the Engineers for research of moving load problem. The simplest case of a moving load investigation is the case of a simple beam over which a concentrated load is moving, that is represented

with a Fourth order *partial* differential equation. This problem has significant effect in civil and mechanical engineering. The dynamic analysis of the vibrating beam is done by neglecting the disconnection of the moving mass from the beam during the motion and result is given by considering mass moving at constant speed and in one direction. Once the load departs from the beam, it begins to vibrate at in free vibration mode. Hence, this process no longer comes within the scope of the experiment.

The problem of moving loads on structures was first considered in the early Nineteenth century when the traversing of bridges by locomotives was analysed, this has been followed by a considerable amount of research on this topic. The purpose of dynamic analysis is to know the structural behaviour under the influence of various loads and to get the necessary information for design such as deformation, moments and dynamic forces etc. Structural analysis is classified in to static and dynamic analysis. Static analysis deals with load which is time independent. But in dynamic analysis magnitude, direction and position of mass change with respect to time. Important dynamic loads for vibration analysis of bridge structure are vehicle motion and wave actions i.e. earthquake, stream flow and winds. Lee (1998) studied extensively the dynamic

responses of a beam acted upon by moving forces or moving masses, in connection with the design of railway tracks and bridges and machining processes. The equation of motion in matrix form has been formulated for the dynamic response of a beam acted upon by a moving mass by using the Lagrangian approach. Convergence of numerical results is found to be achieved with just a few terms for the assumed deflection function.Kargarnovin and Younesian (2005) also analysed the dynamic response of infinite Timoshenko and Euler-Bernoulli beams on nonlinear viscoelastic foundations to harmonic moving loads. Mehri, B. A Davar, &Rahmani O.(2009) presented the linear dynamic response of uniform beams with different boundary conditions excited by a moving load, based on the Euler- Bernoulli beam theory. Using a dynamic green function, effects of different boundary conditions, velocity of load and other parameters are assessed and some of the numerical

results are compared with those given in the references.

Mathematical Formulation

In this section, the dynamic response of a Bernoulli Beam on Winkler foundation under the action of moving partially distributed load is analysed under a non-prismatic Euler-Bernoulli beam of length L resting on a Winkler foundation and traversed by uniform partially distributed moving mass. The resulting vibrational behaviour of this system is described by the following partial differential equations.

$$EIW_{xxx}(x,t) + \frac{m}{L} W_{tt}(x,t) + KW(x,t) = f(x,t)(1.0)$$

Where $f(x,t)$ is the applied moving mass defined as

$$f(x,t) = \left(\frac{1}{\epsilon}\right)\{-Mg - MW_{tt}(x,t)\}\left[H\left(x - \epsilon + \frac{\epsilon}{2}\right) - H\left(x - \epsilon - \frac{\epsilon}{2}\right)\right](1.1)$$

NOMENCLATURE

Parameter	Description
L	Beam length
EI	Flexural rigidity of the beam
Æ	Modulus of Elasticity
Ĝ HÖŦ	The lateral deflection of the beam measured upwards from its equilibrium when unloaded axial coordinate.
Ĝ	Axial coordinate
Ç	The coefficient of Winkler foundation (force per length squared)
Đ	The constant mass per unit length of the beam
Đ	The mass of the load
Ě	The time
Ĵ	The acceleration due to gravity
Ä	Fixed length of the beam



Furthermore, the total derivative $W_t(x,t)$ which appears in equation (1.0) is defined as

$$W_t(x,t) = W_t(x,t) + 2VW_x(x,t) + V^2W_{xx}(x,t) \quad (1.2)$$

Where V is the constant velocity of the moving mass which is defined as

$$\varepsilon = Vt + \frac{\varepsilon}{2} \quad (1.3)$$

$H(x)$ is the Heaviside unit function usually defined as

$$H(x) = \begin{cases} 0, & x < 0 \\ 1, & x > 0 \end{cases} \quad (1.4)$$

1.1 BOUNDARY CONDITIONS

The pertinent boundary conditions for the problem under consideration can be any of the following classical boundary conditions.

$$\left. \begin{aligned} W(x,t) &= W_t(x,t) = 0 & \text{at } x=0 \text{ or } x=l \\ W(x,t) &= W_{tt}(x,t) = 0 & \text{at } x=0 \text{ or } x=l \\ W_{xx}(x,t) &= W_{xxx}(x,t) = 0 & \text{at } x=0 \text{ or } x=l \\ W_x(x,t) &= W_{xxx}(x,t) = 0 & \text{at } x=0 \text{ or } x=l \end{aligned} \right\} \quad (1.5)$$

Finally, the initial conditions are:

$$\left. \begin{aligned} W(x,0) &= 0 \\ W_t(x,0) &= 0 \end{aligned} \right\} \quad (1.6)$$

1.2 SOLUTION OF THE PROBLEM

In this section, we proceed to solve the above initial boundary – value problem described by equation (1.0), (1.1) and (1.6)

To this effect, we assume that the unknown initial deflection,

$$W(x,t) = \sum_{j=1}^{\infty} T_j(t) X_j(x) \quad (1.7)$$

Where $T_j(t)$ are unknown functions of time t and

$$EI \sum_{j=1}^{\infty} T_j(t) X_j^{(iv)}(x) + \frac{m}{L} \sum_{j=1}^{\infty} \ddot{T}_j(t) X_j(x) + K \sum_{j=1}^{\infty} T_j(t) X_j(x) = f(x,t) \quad (1.8)$$

At this juncture, it is remarked that the applied force can also be expressed as a series solution to equation (3.7) then we have

$$f(x,t) = \sum_{j=1}^{\infty} T_{fj}(t) X_j(x) \quad (1.9)$$

Where T_{fj} are unknown functions of time different from those T_j . Equations (1.8) and (1.9) yield

$$\begin{aligned} EI \sum_{j=1}^{\infty} T_j(t) X_j^{(iv)}(x) + \frac{m}{L} \sum_{j=1}^{\infty} \ddot{T}_j(t) X_j(x) + K \sum_{j=1}^{\infty} T_j(t) X_j(x) \\ = \sum_{j=1}^{\infty} T_{fj}(t) X_j(x) \end{aligned} \quad (2.0)$$

It is noted that equation (2.0) has two sets of unknowns viz: the T_j 's and the T_{fj} 's. This naturally makes equation (2.0) highly coupled. To reduce this high degree of coupleness, we would have to determine one of these sets of unknowns. We remark, however, that we find it convenient to determine the T_{fj} 's. To this end, we first notice that equations (1.1) and (1.9) yield

$$\begin{aligned} \sum_{j=1}^{\infty} T_{fj}(t) X_j(x) &= \left(\frac{1}{\varepsilon} \right) \left\{ -Mg \left[H\left(x - \varepsilon + \frac{\varepsilon}{2}\right) - H\left(x - \varepsilon - \frac{\varepsilon}{2}\right) \right] \right. \\ &\quad \left. \left[H\left(x - \varepsilon + \frac{\varepsilon}{2}\right) - H\left(x - \varepsilon - \frac{\varepsilon}{2}\right) \right] \right\} \end{aligned} \quad (2.1)$$

Next, multiply equation (2.1) by the unknown normalized deflection function $X_i(x)$ and then integrate the resulting equation over the length of the beam to obtain

$$\begin{aligned} T_{fj} &= Mg \left[X_i(\varepsilon) + \frac{\varepsilon^2}{24} X_j''(\varepsilon) \right] \\ &- M \sum_{j=1}^{\infty} \ddot{T}_j(t) \left\{ X_j(\varepsilon) X_i(\varepsilon) + \frac{\varepsilon^2}{24} \left[X_j''(\varepsilon) X_i(\varepsilon) + 2X_j'(\varepsilon) X_i'(\varepsilon) \right] \right\} \\ &- 2MV \sum_{j=1}^{\infty} \dot{T}_j(t) \left\{ X_j'(\varepsilon) X_i(\varepsilon) + \frac{\varepsilon^2}{24} \left[X_j'''(\varepsilon) X_i(\varepsilon) + 2X_j''(\varepsilon) X_i'(\varepsilon) \right] \right\} \\ &- MV^2 \sum_{j=1}^{\infty} T_j(t) \left\{ X_j''(\varepsilon) X_i(\varepsilon) + \frac{\varepsilon^2}{24} \left[X_j^{(iv)}(\varepsilon) X_i(\varepsilon) + 2X_j'''(\varepsilon) X_i'(\varepsilon) \right] \right\} \end{aligned} \quad (2.2)$$

Next substituting equation (2.2) into equation (2.0), the approximate governing equation is found to be

$$\begin{aligned} EI \sum_{j=1}^{\infty} T_j(t) X_j^{(iv)}(x) + \frac{m}{L} \sum_{j=1}^{\infty} \ddot{T}_j(t) X_j(x) + K \sum_{j=1}^{\infty} T_j(t) X_j(x) \\ = \sum_{j=1}^{\infty} X_j \left(-Mg \left[X_i(\varepsilon) + \frac{\varepsilon^2}{24} X_j''(\varepsilon) \right] \right. \\ \left. - M \sum_{j=1}^{\infty} \ddot{T}_j(t) \left\{ X_j(\varepsilon) X_i(\varepsilon) + \frac{\varepsilon^2}{24} \left[X_j''(\varepsilon) X_i(\varepsilon) + 2X_j'(\varepsilon) X_i'(\varepsilon) \right] \right\} \right. \\ \left. - 2MV \sum_{j=1}^{\infty} \dot{T}_j(t) \left\{ X_j'(\varepsilon) X_i(\varepsilon) + \frac{\varepsilon^2}{24} \left[X_j'''(\varepsilon) X_i(\varepsilon) + 2X_j''(\varepsilon) X_i'(\varepsilon) \right] \right\} \right. \\ \left. - MV^2 \sum_{j=1}^{\infty} T_j(t) \left\{ X_j''(\varepsilon) X_i(\varepsilon) + \frac{\varepsilon^2}{24} \left[X_j^{(iv)}(\varepsilon) X_i(\varepsilon) + 2X_j'''(\varepsilon) X_i'(\varepsilon) \right] \right\} \right) \end{aligned}$$

To simplify equation (2.3), we noted that for free vibration of an Euler-Bernoulli beam, we have

$$X_j^{(iv)}(x) = \beta_j^4 X_j(x) = 0 \quad (2.4)$$

$$\text{where } \beta_j^4 = \frac{mP_j^2}{EI} \quad (2.5)$$

and P_j^2 is the square of the j^{th} natural frequency of the beam.

For arbitrary $X_j(x)$, we have

$$\begin{aligned} \ddot{T}_j(t) + \left(P_j^2 + \frac{K}{m} \right) T_j(t) \\ = \frac{1}{m} \left(-Mg \left[X_i(\varepsilon) + \frac{\varepsilon^2}{24} X_j''(\varepsilon) \right] \right. \\ \left. - M \sum_{j=1}^{\infty} \ddot{T}_j(t) \left\{ X_j(\varepsilon) X_i(\varepsilon) + \frac{\varepsilon^2}{24} \left[X_j''(\varepsilon) X_i(\varepsilon) + 2X_j'(\varepsilon) X_i'(\varepsilon) \right] \right\} \right. \\ \left. - 2MV \sum_{j=1}^{\infty} \dot{T}_j(t) \left\{ X_j'(\varepsilon) X_i(\varepsilon) + \frac{\varepsilon^2}{24} \left[X_j'''(\varepsilon) X_i(\varepsilon) + 2X_j''(\varepsilon) X_i'(\varepsilon) \right] \right\} \right. \\ \left. - MV^2 \sum_{j=1}^{\infty} T_j(t) \left\{ X_j''(\varepsilon) X_i(\varepsilon) + \frac{\varepsilon^2}{24} \left[X_j^{(iv)}(\varepsilon) X_i(\varepsilon) + 2X_j'''(\varepsilon) X_i'(\varepsilon) \right] \right\} \right) \end{aligned} \quad (2.6)$$

Equation (2.6) is the desired set of coupled second order differential equations. By solving these equations in (2.6) for $T_j(t)$'s and substituting the resulting expression into equation (1.7), the desired solution for the vibration of the beam under different boundary conditions and with any number of modal shapes can be determined.

1.3 SIMPLY-SUPPORTED BEAM

To solve the above coupled equation (2.6), we need to know the exact form of the normalized deflection X_j . As a matter of fact, there exists various turns of X_j depending on the vibrating configurations of the beam. In other words, the solution of equation (2.6) depends on the associated boundary conditions as the exact form of X_j depends on the type of boundary conditions under consideration.

Hence, as an illustrative example, we consider a beam which is simply supported that is a beam whose boundary conditions are given as

$$W(x,t) = 0 = W_{xx}(x,t) = 0 \text{ at } x=0 \text{ and } x=L \quad (2.7)$$

It is well known that for a simply supported beam

$$X_j(x) = \sqrt{\frac{2}{L}} \sin \frac{j\pi x}{L}, \quad j = 1, 2, 3, \dots \quad (2.8)$$

Direct substituting of (2.8) into (2.6) will yield the desired governing equation which is, however an approximate one. It is remarked, that for the configuration under discussion an exact differential governing equation can be derived by going through arguments similar to those used in obtaining equation (2.6).

$$\begin{aligned} \sum_{j=1}^{\infty} T_{fj} \int_0^L \frac{2}{L} \sin \frac{j\pi x}{L} \sin \frac{i\pi x}{L} dx \\ = -\frac{Mg}{\varepsilon} \int_0^L \frac{2}{L} \sin \frac{i\pi x}{L} \left[H\left(x - \varepsilon + \frac{\varepsilon}{2}\right) - H\left(x - \varepsilon - \frac{\varepsilon}{2}\right) \right] dx \\ - \frac{M}{\varepsilon} \left\{ \sum_{j=1}^{\infty} \ddot{T}_j(t) \int_0^L \sin \frac{j\pi x}{L} \sin \frac{i\pi x}{L} \left[H\left(x - \varepsilon + \frac{\varepsilon}{2}\right) - H\left(x - \varepsilon - \frac{\varepsilon}{2}\right) \right] dx \right. \\ \left. + 2V \sum_{j=1}^{\infty} \dot{T}_j(t) \int_0^L \left(\frac{L}{j\pi} \right) \left(\frac{2}{L} \right) \cos \frac{j\pi x}{L} \sin \frac{i\pi x}{L} \left[H\left(x - \varepsilon + \frac{\varepsilon}{2}\right) - H\left(x - \varepsilon - \frac{\varepsilon}{2}\right) \right] dx \right. \\ \left. + V^2 \sum_{j=1}^{\infty} T_j(t) \int_0^L \frac{2}{L} \left(\frac{-L^2}{i^2\pi^2} \right) \sin \frac{j\pi x}{L} \sin \frac{i\pi x}{L} \left[H\left(x - \varepsilon + \frac{\varepsilon}{2}\right) - H\left(x - \varepsilon - \frac{\varepsilon}{2}\right) \right] dx \right\} \end{aligned} \quad (2.9)$$

After integrating equation (2.9) for arbitrary $X_j(X)$ we obtain

$$\ddot{T}_j(t) + \left(P_j^2 + \frac{k}{m} \right) T_j(t) = \frac{1}{m} \left[-\frac{Mg}{j\pi\varepsilon} \sqrt{8L} \sin \left(\frac{j\pi\varepsilon}{L} \right) \sin \left(\frac{i\pi\varepsilon}{L} \right) \sum_{j=1}^{\infty} T_{fj} \right]$$

$$\begin{aligned}
& -\frac{2M}{\epsilon\pi} \sum_{j=1}^{\infty} \ddot{T}_j(t) \frac{1}{(i-j)} \left\{ \sin\left(\frac{j\pi\epsilon}{2L}\right) \cos\frac{\pi\epsilon}{L}(i-j) \right\} \\
& + \frac{2M}{\epsilon\pi} \sum_{j=1}^{\infty} \ddot{T}_j(t) \frac{1}{(i+j)} \left\{ \cos\frac{\pi\epsilon}{L}(i+j) \sin\frac{\pi\epsilon}{2L}(i+j) \right\} \\
& + \frac{2MV}{\epsilon} \sum_{j=1}^{\infty} \dot{T}_j(t) \sqrt{\frac{2}{L}} i \left\{ \sin\frac{\pi\epsilon}{L}(i+j) \sin\frac{\pi\epsilon}{2L}(i+j) \right\} \\
& + \left[\sin\frac{\pi\epsilon}{L}(i-j) \sin\frac{\pi\epsilon}{2L}(i-j) \right] \left\{ \right\} \\
& + \frac{MV^2}{\epsilon} \left(\frac{i\pi}{L} \right) \sum_{j=1}^{\infty} T_j(t) \sqrt{\frac{2}{L}} \frac{1}{(i+j)} \left\{ \left[\sin\frac{\pi\epsilon}{L}(i+j) \cos\frac{\pi\epsilon}{2L}(i+j) \right] \right\} \\
& + \left[\cos\frac{\pi\epsilon}{L}(i+j) \sin\frac{\pi\epsilon}{2L}(i+j) \right] \quad j = 1, 2, 3, \dots \quad i \neq j \quad (3.0)
\end{aligned}$$

We now use the finite difference method in order to solve the above equation numerically, we made use of approximate central difference method, we obtain.

$$\begin{aligned}
& \left\{ m + \frac{2M}{\epsilon\pi} \left[\frac{1}{(i-j)} \cos\frac{\pi\epsilon}{L}(i-j) \sin\frac{\pi\epsilon}{2L}(i-j) - \frac{1}{(i+j)} \cos\frac{\pi\epsilon}{L}(i+j) \sin\frac{\pi\epsilon}{2L}(i+j) \right] \right\} T_{j+1} + \\
& \frac{hMV}{\epsilon} \sqrt{\frac{2}{L}} \left[\frac{1}{(i+j)} \sin\frac{\pi\epsilon}{L}(i+j) \sin\frac{\pi\epsilon}{2L}(i+j) + \frac{1}{(i-j)} \sin\frac{\pi\epsilon}{L}(i-j) \sin\frac{\pi\epsilon}{2L}(i-j) \right] T_{j+1} + \\
& \left\{ -2M - \frac{4M}{\epsilon\pi} \left[\frac{1}{(i-j)} \cos\frac{\pi\epsilon}{L}(i-j) \sin\frac{\pi\epsilon}{2L}(i-j) + \frac{1}{(i+j)} \cos\frac{\pi\epsilon}{L}(i+j) \sin\frac{\pi\epsilon}{2L}(i+j) \right] + K + \right. \\
& m h^2 P_j^2 + \frac{h^2 MV^2 i\pi}{\epsilon L} \sqrt{\frac{2}{L}} \left[\frac{1}{(i-j)} \sin\frac{\pi\epsilon}{L}(i-j) \cos\frac{\pi\epsilon}{2L}(i-j) - \frac{1}{(i+j)} \cos\frac{\pi\epsilon}{L}(i+j) \sin\frac{\pi\epsilon}{2L}(i+j) \right] + \\
& \left. j) \right\} T_j + \left\{ M + \frac{2M}{\epsilon\pi} \left[\frac{1}{(i-j)} \cos\frac{\pi\epsilon}{L}(i-j) \sin\frac{\pi\epsilon}{2L}(i-j) - \frac{1}{(i+j)} \cos\frac{\pi\epsilon}{L}(i+j) \sin\frac{\pi\epsilon}{2L}(i+j) \right] + \right. \\
& \frac{hMV}{\epsilon} \sqrt{\frac{2}{L}} \left[\frac{1}{(i+j)} \sin\frac{\pi\epsilon}{L}(i+j) \cos\frac{\pi\epsilon}{2L}(i+j) + \frac{1}{(i-j)} \sin\frac{\pi\epsilon}{L}(i-j) \cos\frac{\pi\epsilon}{2L}(i-j) \right] \left. \right\} T_{j-1} = \\
& -\frac{h^2 Mg}{i\pi\epsilon} \sqrt{8L} \sin\left(\frac{j\pi\epsilon}{L}\right) \sin\left(\frac{i\pi\epsilon}{L}\right) \quad (3.1)
\end{aligned}$$

NUMERICAL RESULTS

The result obtained in equation (3.1) for nonlinear dynamical systems subjected to partially distributed load is discussed in this chapter using analytical-numerical method. Which made use of approximate finite difference method and MATLAB was used for the values of the variable used and the following graphs were plotted as shown in figures 1.1- 1.8.

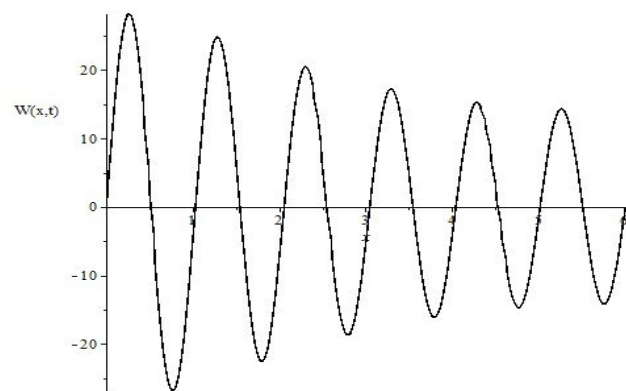


Fig 1.1: Graph of Deflection against distance at K = 2

It is found that as the value of deflection increases and the value of distance increases there is a decrease in velocity deflection at K=2.

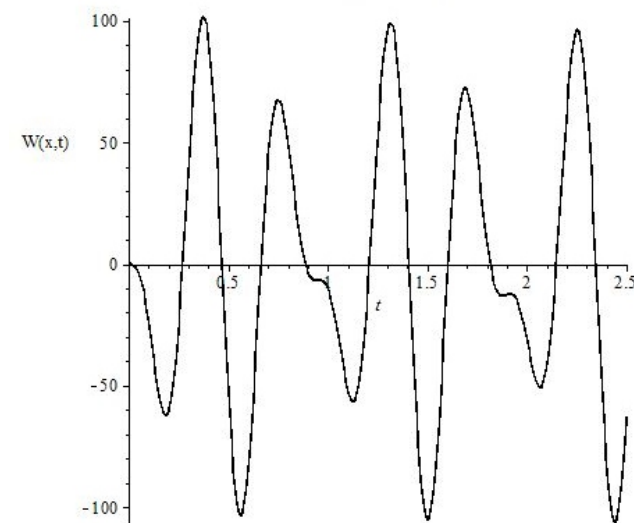


Fig 1.2: Graph of Deflection against time at K = 2

It is found that at a constant value of time (t) the deflection increases and also decreases at k=2.

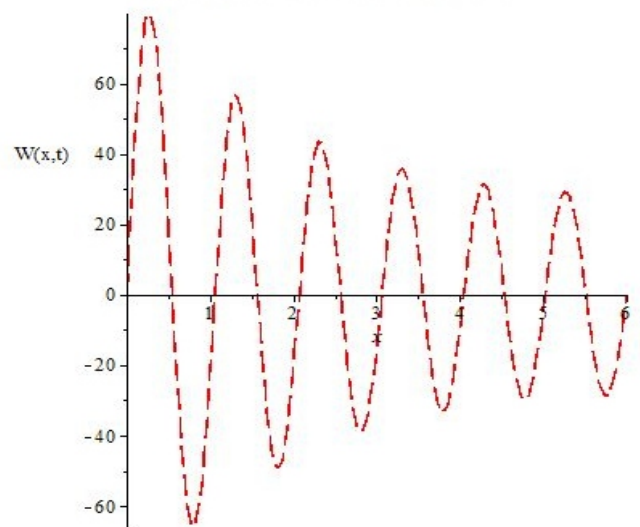


Fig 1.3 Graph of deflection against distance at K = 4

It is found that as the value of deflection increases and the value of distance increases there is a decrease in velocity deflection at K=4.

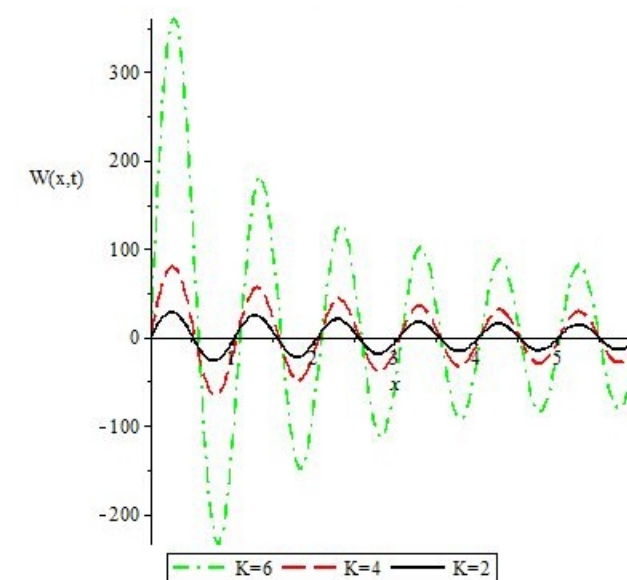


Fig 1.4: Graph of Deflection against distance at various values of K.

Deflection against distance at various values of K, which shows that at a constant value of velocity deflection increases as the value of K increases. This was in agreement with the existing result.

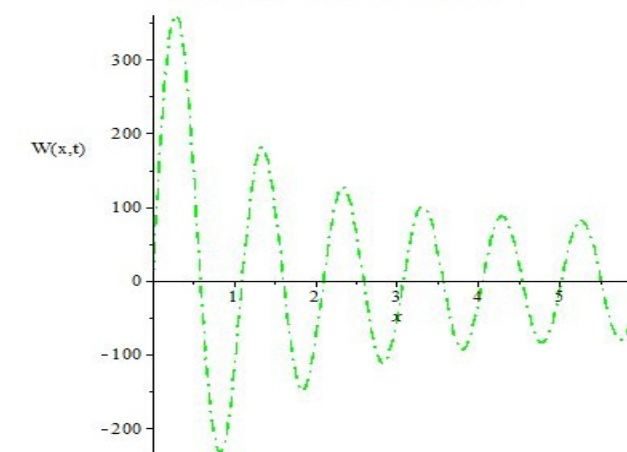


Fig 1.5: Graph of deflection against distance at K = 6.

is found that deflection decreases against distance at a constant value of x at K=2.

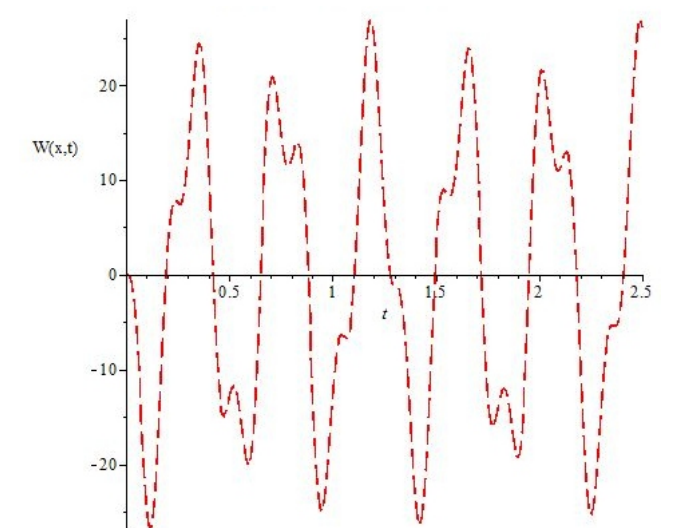


Fig 1.6: Graph of deflection on against time at K = 4.

Deflection against time, it was found that the deflection decreases first and later increases at different time (t) when K=4

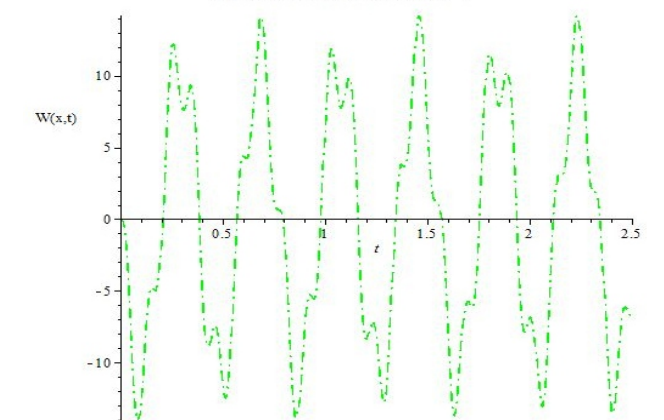


Fig 1.7: Graph of deflection against time at K = 6.

Deflection against time, it was found that the deflection decreases first and later increases at different time (t) which was the same result we are given in Fig 1.6.

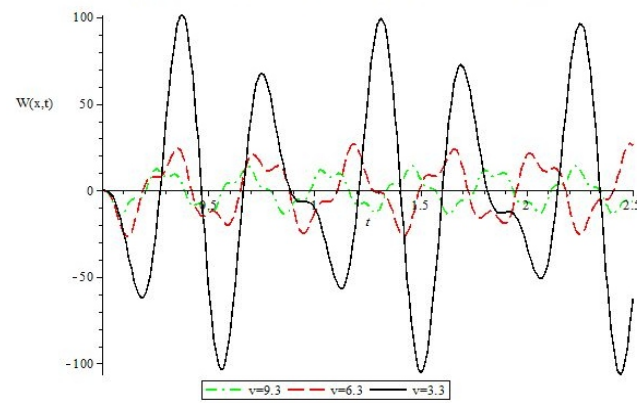


Fig 1.8: Graph of deflection against time when $K = 2$ at various values of velocity

Deflection against time When $K=2$ at various values of velocity $V= 9.3$, $V=6.3$ and $V=3.3$ at constant value of K deflection reduces as velocity increases.

NUMERICAL DISCUSSION

The dynamic response of nonlinear dynamical systems were considered. Also the systems were subjected to partially distributed load is observed for various values deflections against time, at constant value of K deflection reduces as velocity increases and also for deflection against distance, at constant value of velocity deflection increases as the value of K increases. The results obtained were compared with the existing result.

REFERENCES

- Leissa A.W. (1989); Closed form exact solutions for the steady state vibrations of continuous systems subjected to distributed exciting forces, *Journal of sound and vibration* 134 pg. 435- 453.
- Achenbach, J. D. & Sun, C. T. (1965): Moving load on a flexibly supported Timoshenko beam, *International Journal of Solids and Structures* 1,353-370.
- Adel A. Al-Azzawi (2011): Analysis of Timoshenko Beam resting on a nonlinear compressional and frictional Winkler foundation: ARPNI, *Journal of Engineering and Applied Sciences. Asian Publishing Network (ARPNI)*. Vol 6. No 11. Pp: 1 – 14.
- Arturo O. Cifuentes (1989): Dynamic response of a beam excited by a moving mass. Beam to uniform Partially Distributed moving loads, *Finite Elements in Analysis and Design* 5,237-246.
- Dehestani, M. Mofid, A. Vafai (2009): Investigation of critical influential speed for moving mass problems on beams, *Applied Mathematical Modeling* 33,3885-3895
- Esmalzadeh E. and Ghorashi 1997: Vibration analysis of Timoshenko Beams traversed by uniform partially distributed moving masses. *Proceeding of the international conference in Engineering Application of Mechanics Tehran* 2,238 - 323.
- Foda, M.A. & Abduljabbar, Z. (1997): *Journal of Sound and Vibration* 230,493-506. A dynamic Green function formulation for the response of a beam structure to a moving mass.
- Fryba, L. (1972): "Vibration of Solids and Structures under Moving Loads", Noordhoff International, Groningen.
- Gbadeyan J.A. and Oni S.T. (1992): Dynamic Response of Moving Concentrated Masses of Elastic Plates on a non-Winkler Elastic Foundation. *Journal of Sound and Vibration* 154(2): 343 - 358.
- Gbadeyan J.A. & Oni S.T. (1995): Dynamics behaviour of Beams and rectangular plates under moving loads. *Journal of Sound and Vibration*. 182 (5) pp. 677 – 695.
- Lueschen, G.G.G, Bergman & Macfarland, D.M. (1996): Green functions for uniform Timoshenko Beams, *Journal of sound and vibration* 194,93-102.
- Michaltsos, G.T. & Kounadis, A.N. (2001): The Effects of Centripetal and Coriolis Forces on the Dynamic Response of Light Bridges Under Moving Loads, *Journal of Sound and Vibration* 247,261-277.
- Gbolagade, A.W ,Gbadeyan, J.A., S.O.S. Olabamiji, O. Otolurin (2003): Response of Elastically Connected Beams subjected to moving forces .*Nigeria Journal of Mathematics and Applications*, 16: 67-79.
- Hankun, Wang and Goong Chen (1991): Asymptotic locations of Eigen frequencies of Euler-Bernoulli Beam with nonhomogeneous structural and viscous damping coefficients. *Journal of control and optimization*. Vol. 29.No. 2. Pp. 347 – 367.
- J.W. Nicholson, L.A Bergman (1986); Free vibration of combined dynamical systems , *American Society of Civil Engineers Journal of Engineering Mechanics* 112 pg. 1- 13.
- Jia-Jang Wu, A.R. Whittaker, M.P. Cartmell (2000): The use of finite element techniques for calculating the dynamic response of structures to moving loads, *Computers & Structures* 78, 789-799.
- Kargarnovin M. H. & Younesian, D. (2005): Response of beams on nonlinear viscoelastic foundations to harmonic moving loads, *Computers & Structures*, 83 1865-1877.
- Kargarnovin, M. H. & Younesian, D. (2004) : Dynamics of Timoshenko beams on Pasternak foundation under moving load, *Mechanics Research Communications*, 31, 713-723.
- L. A Bergman, J.W. Nicholson (1985); Forced vibration of a damped combined linear system, *American Society of Mechanical Engineers , Journal of vibration, Acoustics, Stress, and Reliability in Design* 107 pg. 275 - 281.
- L. Fryba (1972); *Vibration of Solids and Structures under Moving loads*, Noordhoff International, Groningen.
- L.A. Bergman, D.M McFarland (1988); On the vibration of a point – supported linear distributed systems, *American Society of Mechanical Engineers, Journal of Vibration, Acoustics, Stress, Reliability in design* 110 pg. 458 – 592.
- Lee, H. P. (1998) : Dynamic response of a Timoshenko beam on a Winkler foundation subjected to a moving mass, *Applied Acoustics*, 55 (3) 203-215.
- M. Gurgoze, H. Erol (2001); Determination of the frequency response function of a cantilevered beam simply supported in-span, *Journal of sound and vibration* 247, pg 372 – 378.
- M.A Foda, Z. Abduljabbar (1998); A dynamic Green function formulation for the response of a beam structure to a moving mass, *Journal of sound and vibration* 210 pg. 295 - 306
- Mehri, B. A Davar, & Rahmani O. (2009): Dynamic Green Function Solution of Beams Under a Moving Load with Different Boundary Conditions. *Scientia Iranica* 16,273-279.
- Michaltos G., Sopianopoulos D., & Kounadis A.N (1996) : The effect of moving mass and other parameters on the dynamic response of a simply supported beam, *Journal of sound and Vibration* 191,357-362.
- Michaltsos, G.T. (2001): *Journal of Sound and Vibration* 258,359-372. Dynamic behavior of a single-span beam Subjected to loads moving with variable speeds.
- Mohan Charan Sethi (2012) ; Dynamic response under moving mass, a thesis submitted in partial fulfillment of the requirement for the degree of bachelor of technology in mechanical Engineering, Department of Mechanical engineering, National institute of technology, Rourkela.
- Muhammed Uncuer, Bozidar Marinkovic and Hur Koser (2007): Simulation of damped – free and damped-damped microbeams dynamics for nonlinear mechanical switch application: Except from the proceeding of the COMSOL, Conference 2007.
- Ojih, P.B., Ibiejugba M.A. and Adejo B.O. (2014): Dynamic Response under moving concentrated loads of uniform Rayleigh beam resting on Pasternak foundation. *International Journal of Scientific and Engineering Research: Volume 5, Issue 6: 1 – 21*.
- Piotri Kozioł and Zdzisław Hryniewicz (2012): Dynamic Response of a beam resting on a nonlinear foundation to a moving load: Coiflet - based solution shock and vibration 19:995 – 1007.

- R. Hamada (1981); Dynamic analysis of a beam under a moving force; a double Laplace transform Solution, Journal of Sound and Vibration 74 pg. 221 – 233.
- S. Kukla, (1997); Application of Green function in frequency analysis of Timoshenko beams with oscillators, Journal of Sound and vibration 205 pg. 355 – 363.
- S. Kukla, B. Posiadala (1994); Free vibration of beams with elasticity mounted masses, Journal of sound and vibration 175 pg. 557 – 564.
- Sadiku S. and Leipholz H.E. (1987): On the dynamics of elastic systems with moving concentrated masses. IngenieurArchiv 57, 223 – 224.
- Seong-Min Kim, (2004): Vibration and Stability of axial loaded beams on elastic foundation under moving harmonic loads, Engineering Structures, 26, 5-105.
- Siddiqui, S.A.Q., Golnaraghi, M.F. &Heppler (2003): Large free vibrations of beam carrying a moving mass. International journal of Nonlinear Mechanics 38, 1481-1493.
- Stansic M.M. and Hardin J.C. (1969): On the response of beams to arbitrary number of concentrated moving masses. Journal of the Frankline Institute 287, 115 – 123.
- T.H Broome Jr. (1989) ; Economical analysis of combined dynamical systems, American Society of Civil Engineers, Journal of Engineering Mechanics 115 pg. 2122 – 2135.
- Thambiratnam, D.&Zhuge, Y. (1996): Dynamic analysis of beams on an elastic foundation subjected to moving loads, Journal of Sound and Vibration, 198, 49-169.
- Timoshenko S. (1927): Vibration of bridges, Transactions of the American society of Mechanical Engineers. 53 – 61.
- Ugural A.C. (1981): Stress in plates and shells. New York, McGraw – Hill.
- UmianEssendemir (2009): Derivation of Equations for Flexure and Shear Deflections of Simply Supported Beams. Vol 6. 187 – 193.

GREEN COMPUTING THROUGH TELECOMMUTING

¹Raji-Lawal Hanat Y., ²Adesina Ademola O., ³Akerele Olubunmi C.

^{1,3}Lagos State University, Ojo, Lagos, Nigeria,

²Olabisi Onabanjo University, Ago-Iwoye, Nigeria

Corresponding author: halaw313@yahoo.com

ABSTRACT

Green computing is a technology that tends towards the sustainability of the environment, through energy efficiency, electronic waste reduction, virtualization, employing thin client, remote administration, green power administration and telecommuting. Commuting entails movement from one point to another, with the aim of satisfying individual needs. The population of people all around the world keeps growing exponentially and the major means of transportation is by road using specifically motor vehicles and sometime locomotive trains which exhaust is Carbon Monoxide (CO). CO has been categorized as a harmful substance to the surrounding, and thus creates more challenges to global warming. In lieu of this, telecommuting has been identified as a major weapon to control the challenge. This technology is a product of information communication technology, specifically the e-commerce. With this technology, the rate at which commuters travel is drastically reduced, thus the rate of deposition of CO to the environment is correspondingly reduced, and thus paving way for a greener environment. This concept termed telecommuting is embraced in this research by introducing e-commerce to a livestock production farm, and software engineering models were employed to design a reliable on-line shopping for a farm. In the findings, the adoption of this technology by clients and farm workers in the livestock farm has reduced foot print on this axis through the technology of telecommuting. This in turn reduced the rate of deposition of CO to the atmosphere.

Key words: Green Computing, Telecommuting, Carbon Foot Print, Population.

Accepted Date: 19 September 2017

INTRODUCTION

As the world is constantly evolving, and there is an improvement in information technology industry, the electronic energy and machine usage will be something to reckon with in the coming years. In recent years, companies in the computer industry have come to realize that going green is in their best interest, both in terms of public relations and reduced costs. The deposition of carbon monoxide (CO) to the atmosphere from locomotive engine and other electronic items usage will the world tend towards an uncomfortable zone, and something to worry about. The deed or thought of how to reduce global warming and hot climate change caused by this machines increase has become impatient. Various machines at different specifications are built daily to tackle and reduce human stress and increase productivity in services rendered. This process requires a larger amount of energy

(power) and money for its effective functioning. It is amazing to know that these are forms of achievement from the ancient days when things are done manually, in the modern days, works are done quickly, everything is working effectively and there is less time to worry about low productivity because the improvement in this machine has aided productivity which is a good achievement. There is a neglect to acknowledge this type of achievement, and the effect it has on the general environment, what it has on the air (for respiration), the food (for consumption) which affects our life in one way or the other. The various chemical substances emitted by these machines i.e. cars, tractors used in farm or other machineries have negative reactions on lives and environment. Therefore there is a need to find for a new measure to curtail this, hence the approach of green computing technology. Green computing also called green technology is