NUMERICAL INVESTIGATION OF SIGNAL AMPLIFICATION VIA VIBRATIONAL RESONANCE IN CHUA'S CIRCUIT

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ABSTRACT

In this paper, we numerically investigated the occurrence of Vibrational Resonance in a modified Chua's oscillator with a smooth nonlinearity, described by a cubic polynomial. Response curves generated from the numerical simulation at the low frequency reveal that the system's response amplitude could be controlled by modulating the conductance parameter of the Chua's circuit, rather modulating the parameters of the fast-periodic force. Modulating the frequency of the fast-periodic force slightly reduces the response amplitude; shifts the peak point to a higher value of the amplitude of the fast-periodic force by widening the resonance curves. Within certain parameter regime of the high frequency ($\Omega \geq 100\omega$),
the system's response gets saturated, and further increase does not affect its amplitude.
Keywords: Chua's circuit, Cubic nonlinearity, Vibrational resonance, Phase portrait

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INTRODUCTION

The study of nonlinear dynamical systems subjected to external periodic forces led to several fascinating phenomena. Such extremely interesting phenomena include resonance with varieties depending on the type of external forces. Stochastic resonance is one such phenomenon in which enhancement of the amplitude of a signal is observed at an optimum noise intensity (Gammaitoni *et al.,* 1998; Li and Zhu, 2001; Casado-Pascual *et al.,* 2005; Li, 2011).Resonance induced by an external noise in the absence of any external periodic force is termed Coherence Resonance (Gang *et al.,* 1993; Pikovsky and Kurths 1997). Chaotic resonance is obtained when one of the driving forces is chaotic in nature (Chew *et al.,* 2005; Nobukawa *et al.,* 2017). Ghost resonance is a type of resonance induced by a multi-frequency force at a frequency absent in the external force (Buld *et al.,* 2003, 2005; Balenzuela *et al.,* 2007; Gomes *et al.,* 2012; Rajasekar and Sanjuan, 2016).More so, when a biharmonic external periodic force drives a nonlinear dynamical system with two different frequencies, say, ω and Ω, such that $\Omega \gg \omega$, it results to the phenomenon of vibrational resonance (Landa and McClintok 2000; Gitterman 2001; Blekhman and Landa 2004; Jeyakumari *et al.,* 2009; Roy-Layinde *et al.,* 2016).

Specifically, in VR the amplitude of the output signal increased from a small value, reached a maximum at one or two critical values of the control parameter and then decayed totally or partially, approaching zero. Since a relatively high-frequency force induces this phenomenon at the low frequency of the input signal, its application spans almost all fields of life such as biology, physics, engineering and medical sciences. Following the pioneering study of Landa and McClintock which shed light on the phenomenon, there has been diverse research of the occurrence of VR in damped, excitable systems or bistable oscillators (Landa & McClintock, 2000; Gitterman, 2001; Blekhman and Landa 2004; Jeyakumari *et al.,* 2009; Rajasekar *et al.,* 2011, 2012; Daza *et al.,* 2013; Chizhevsky, 2014; Deng *et al.,* 2014; Abirami *et al.,* 2017).

Also, the ubiquitous nature of VR makes it applicable across multiple fields such as neuroscience, laser physics, ionospheric physics, acoustics, plasma physics and atomic physics (Rajasekar and Sanjuan, 2016). For instance, in acoustics, VR could be used in control and enhancement of signal propagation in complex networks and in the design of laser and telecommunication devices (Yang and Liu, 2010). Experimental investigation of VR has been reported in analog circuit simulation of an excitable system (Baltanás *et al.,* 2003) and in a vertical cavity surface emitting laser system (Chizhevsky and Giacomelli, 2006, 2008).

More recently, VR occurrence was studied experimentally in a nonlinear electronic circuit of great significance in the field of nonlinear science - Chua's circuit (Chua, 1994). It is well known from nonlinear sciences that the Chua's circuit is commonly used as a prototype circuit for investigating a variety of dynamics (Matsumoto *et al.,* 1985; Jothimurugan *et al.,* 2013).

In this direction, we numerically investigate the occurrence of VR in a modeled Chua circuit, with cubic nonlinearity. The system is a simplest modeled electrical circuit which presents chaotic behavior and can be described with a third-degree autonomous differential equation. The two main features are the fact that it is a third order system and that it's also nonlinear. For instance, in Chua's circuit, the nonlinearity and the local activity are given by a single nonlinear element which is often called Chua's diode. The only known thorough proof of chaotic attractors in a Chua's circuit depends specifically on the type of a piecewise-linear nonlinearity. This accounted for piecewise-linear function of Chua's circuit when he invented this circuit (Chua, 1992).

The outline of the paper is as follows: In section 2, the mathematical model of the Chua's circuit is given and the response of the system at low frequency (ω) is evaluated numerically in section 3. The dependence of the Response amplitude on the Amplitude of the high frequency driven force is then presented in section 4. Based on the results in section 4, other numerical cases of the occurrence of VR were discussed in the same section. Finally, a conclusion is given in the last section.

THE MODEL

As the starting point, we present the well-studied Chua's circuit modeled as a dimensionless dynamical system of in the form;

$$\dot{x} = a(y - x + f(x)), \quad \dot{y} = x - y + z, \quad \dot{z} = by \qquad (1)$$

Here $f(x)$ represents the piece-wise linear function,

$$f(x) = cx + \frac{1}{2}(d - c)(|x+1| - |x-1|) \qquad (2)$$

Where c and d are constant parameters. The system appears not to be mathematically elegant compared with other nonlinear systems. However, a more elegant variant equation with unnecessary terms removed and others normalized to 1.0 was modeled. Recently, an elegant version of Eqn (1) and (2) was modeled by replacing the piece-wise linear function with a nonlinear function

$$\dot{x} = ay + x - x^3, \quad \dot{y} = x + z, \quad \dot{z} = by \qquad (3)$$

The modeled system was reported to be chaotic for $a=0.3$ and $b=-1$ with its dynamics mimicking the well-known Chua's circuit.

To investigate the occurrence of vibrational resonance, the system is driven by abi-harmonic periodic force of the form $f\cos\omega t$ and $g\cos\Omega\tau$ with $\Omega >> \omega$, such that

$$\dot{x} = ay + x - x^3 f\cos\omega t + g\cos\Omega\tau, \quad \dot{y} = x + z, \quad \dot{z} = by \qquad (4)$$

where $f\cos\omega t$ is a low frequency driven force, and $g\cos\Omega\tau$ the high frequency driven force.

NUMERICAL SIMULATIONS

To investigate the occurrence of VR in the system, we numerically integrate the three coupled autonomous ordinary differential equations (ODEs) given in Eqn. (4).

These equations were integrated using the Fourth-Order Runge-Kutta (FORK) scheme with fixed step sizes Δt=0. 1over a simulation time interval T_s = nT with $T = \frac{2\pi}{\omega}$ being the period of the harmonic oscillation and number of complete oscillation n=20 was performed.. The following parameters were fixed throughout the paper $a=0.3$, $b=-1$ and $\omega=1$. The initial conditions are

$$x(0) = 0, y(0) = -3 \text{ and } z(0) = 1$$

To analyse vibrational resonance, we use the response amplitude Q, obtained from the Fourier spectrum of sine and cosine component, Q_s and Q_c respectively at the slow signal frequency ω.

$$Q_s = \frac{2}{nT}\int_0^{nT} \chi(t)\sin\omega t\,dt \qquad (5)$$

$$Q_c = \frac{2}{nT}\int_0^{nT} \chi(t)\cos\omega t\,dt \qquad (6)$$

The response amplitude $$Q = \frac{\sqrt{Q_s^2 + Q_c^2}}{F} \qquad (7)$$

The VR effect can be quantified by plotting the response Q of the system against the amplitude or frequency of the high frequency driven force g. The response of the circuit displays resonance when the parameter g or Ω of the high-frequency force is varied.

RESULTS AND DISCUSSION

Results were presented in the figures below, and each figure was computed by varying at least one of the parameters of the system. Figure1 (a) - (f) and figure 2(a) – (f) depicts the phase portraits and time evolution of the system with varying frequency of the fast driven force Ω. The attractors obtained are close to chaotic attractors reported by Tsuneda *et al.* (2005) for Chua's oscillator with a cubic nonlinearity, but with little change in its trajectory. The effects of frequency of the fast, driven force on the trajectory shift the periodic and chaotic attractors toward the equilibrium. Increasing the frequency from 5ω in figure 1(a) and 2(a) to 25ω in figure 1(b) and 2(b), the attractor moved from being a single chaotic attractor to two periodic attractors and spending approximately same time in the negative and positive halves. A further increase in the frequency of the fast, driven force 1(c-f) and 2(c-f) repeats the trajectory of the system, showing that the motion of the attractors of the system is periodic within a parameter regime of the fast periodic force.

Q

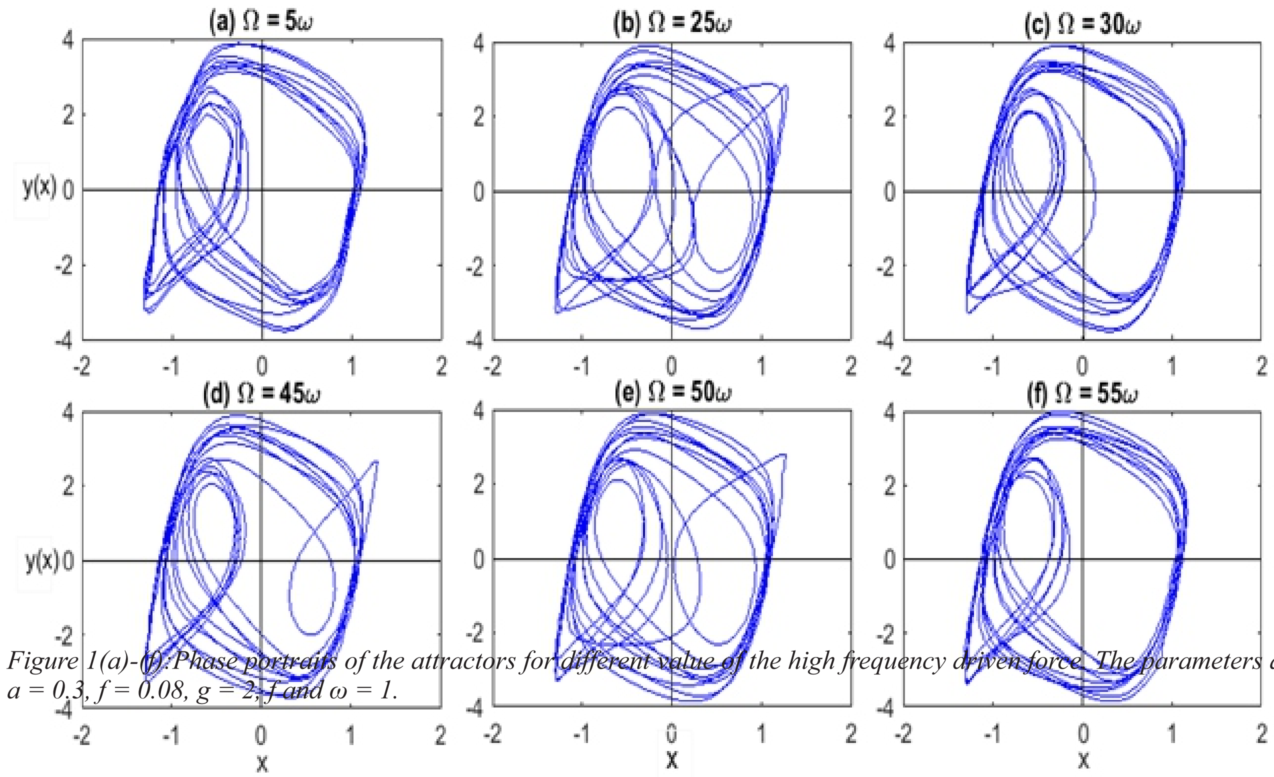

Figure 1(a)-(f): Phase portraits of the attractors for different value of the high frequency driven force. The parameters are a = 0.3, f = 0.08, g = 2, f and ω = 1.

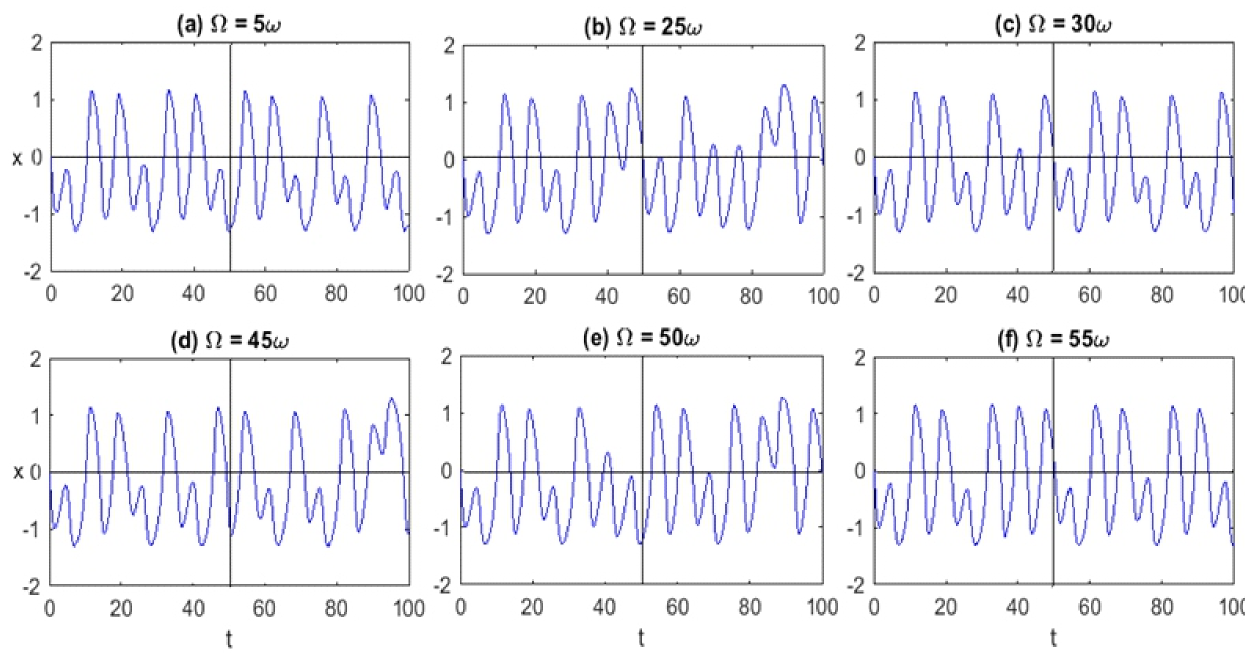

Figure 2(a)-(f): Time evolution of the attractors for different high frequency. The corresponding parameters are the same as those in figure 1.

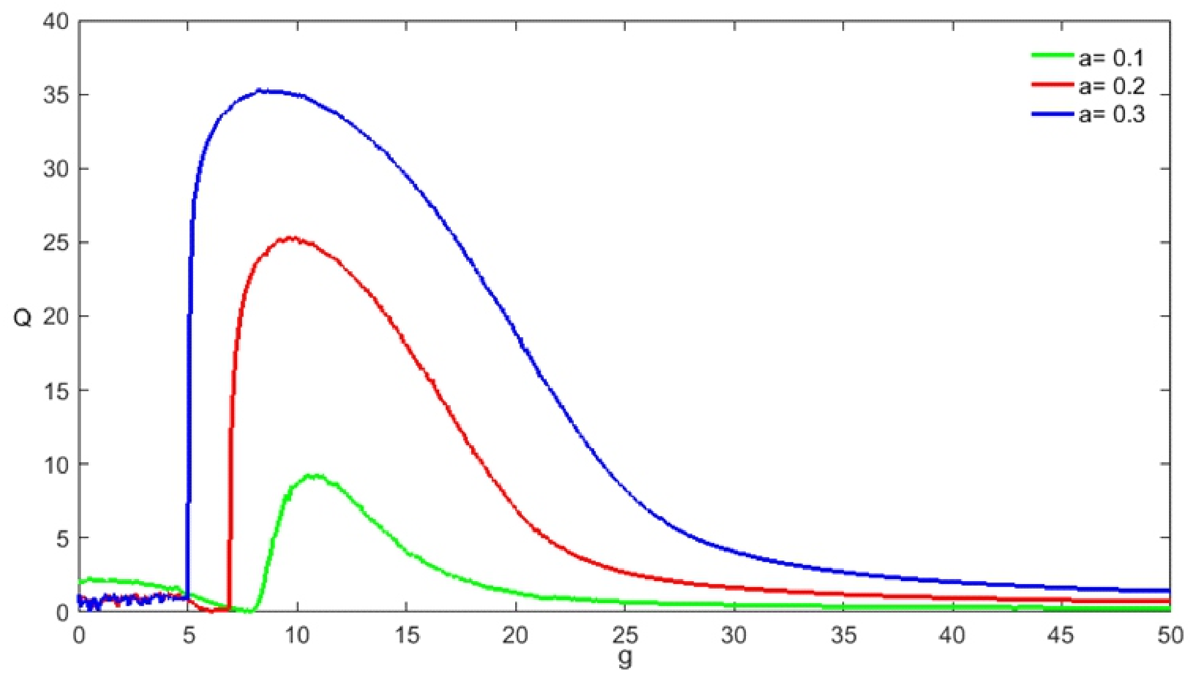

Figure 3:Superimposed response plot of Q against g, for different values of a =0.1, 0.2 and 0.3, with other parameters fixed at f = 0.08, ω = 1 and Ω = 10ω.

In Figure 3, the dependence of the response amplitude Q on the amplitude of the high frequency driven force g is illustrated for different values of the conductance in the Chua's circuit. Under this condition, the VR phenomenon observed system is a bit different from the traditional VR theory; the VR phenomenon is found and observed via modulating the parameter of the conductance parameter in the system instead of changing the frequency or amplitude of the fast driven force.

Increasing the magnitude of the parameter increases the system's response greatly. As observed from the superimposed response plot in Figure 3, the maximum response ($Q=35.26$), corresponds to $a = 0.3$ which was reported by Tsuneda *et al.* (2005) as a chaotic value of the parameter in the Chua's oscillator with cubic nonlinearity.

To further characterize the VR phenomenon observed, the response amplitude of the system is plotted in Figure 4 as a function of the amplitude of the fast driven force with varying values of Ω. By comparing figure 3 and figure 4, one can obviously see that the first plot in Figure 4 with $\Omega = 10\omega$ corresponds to response plot in figure 3 for $a=0.3$. We shall look closely at the occurrence of VR in this regime throughout the analysis since this happens to be the point at which the system responds greatly and with Q maximum. A further increase in the frequency of the fast, driven force reduces the response amplitude but shifts the resonance point outward to higher values of g. The maximum response is observed at $g = 8.70$ and others at $g = 17.10, 39.50,$ and 74.00 corresponding to the response amplitude $Q = 34.32, 33.61$ and 33.13 respectively. The values agree with the experimental investigation of the occurrence of VR in a Chua's circuit (Jothimurugan *et al.,* 2013). The outward shift and increase in width of the resonance curve observed here can be attributed to the effect of the high frequency of the fast signal as observed in on the motion of the attractors of the system. Taking the frequency of the fast periodic force for one complete cycle of the motion of the attractors as $\Omega = 50\omega$. In this case, it is worth noting the fact that the difference in response amplitude for the first cycle of the motion is approximately $Q_{20\omega} - Q_{10\omega} = 0.94$, with 8.30 as the difference in g. In that respect, the motion in the second cycle shows that the higher value of the frequency of the fast periodic force (i. e $\Omega > 50\omega$), the motion of the attractor moves towards beginning another cycle, such as in figure 1(f). The response amplitude corresponding to this can be traced to the two response curves in figure 4 for $\Omega = 50\omega$ and 100ω showing a very little variation in the response amplitude $Q_{50\omega} - Q_{100\omega} = 0.48$, with g difference of 34.50.

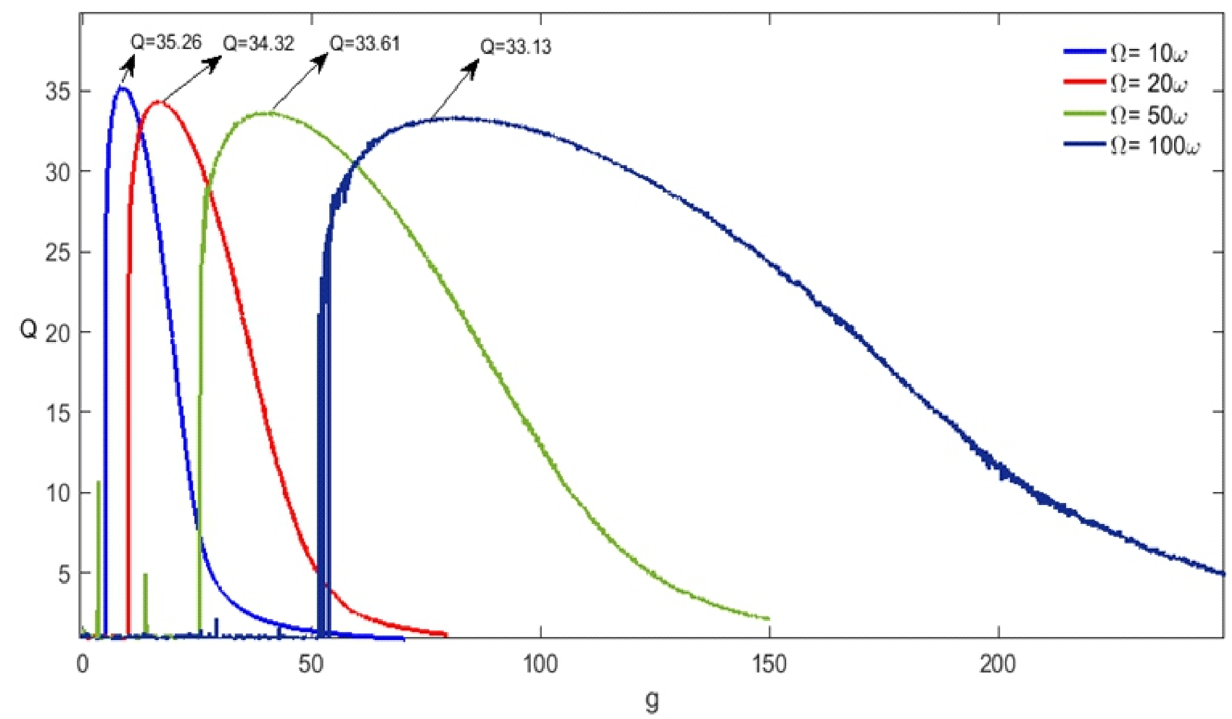

Figure 4:Superimposed response plot of Q against g, for different values of $\Omega=10\omega, 20\omega, 50\omega$ and 100ω, with other parameters fixed at $f = 0.08$, $\omega = 1$ and $a = 0.3$.

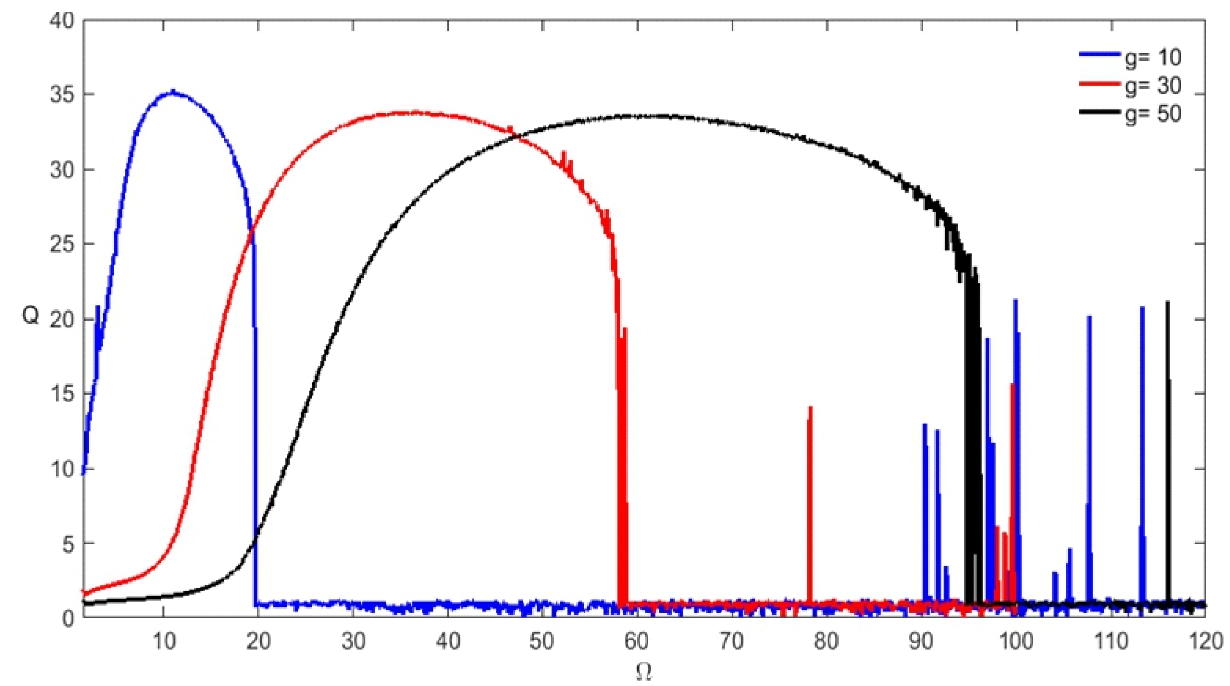

Figure 5: Superimposed response plot of Q against Ω, for different values of g =*10*,*30* and *50,* with other parameters fixed, $f = 0.08$, $\omega = 1$ and $a = 0.3$.

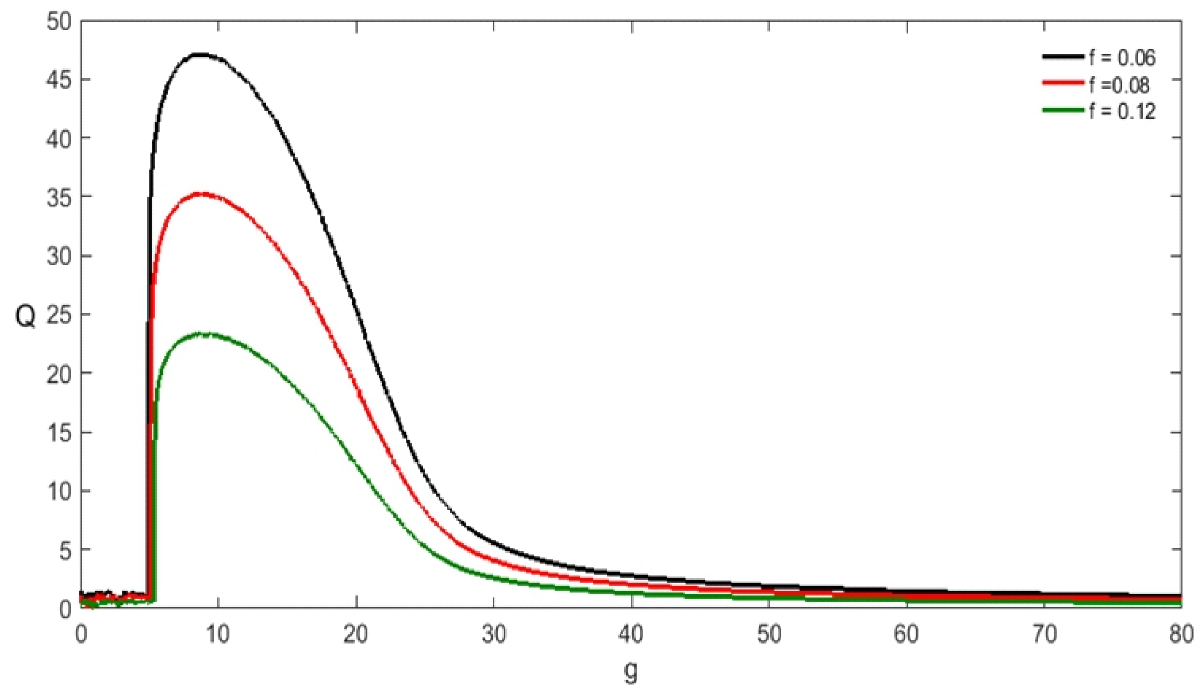

Figure 6: Superimposed response plot of Q against g, for different values of f=0.06, *0.08* and *0.12,* with other parameters fixed, Ω= 10ω, $\omega = 1$ and $a = 0.3$.

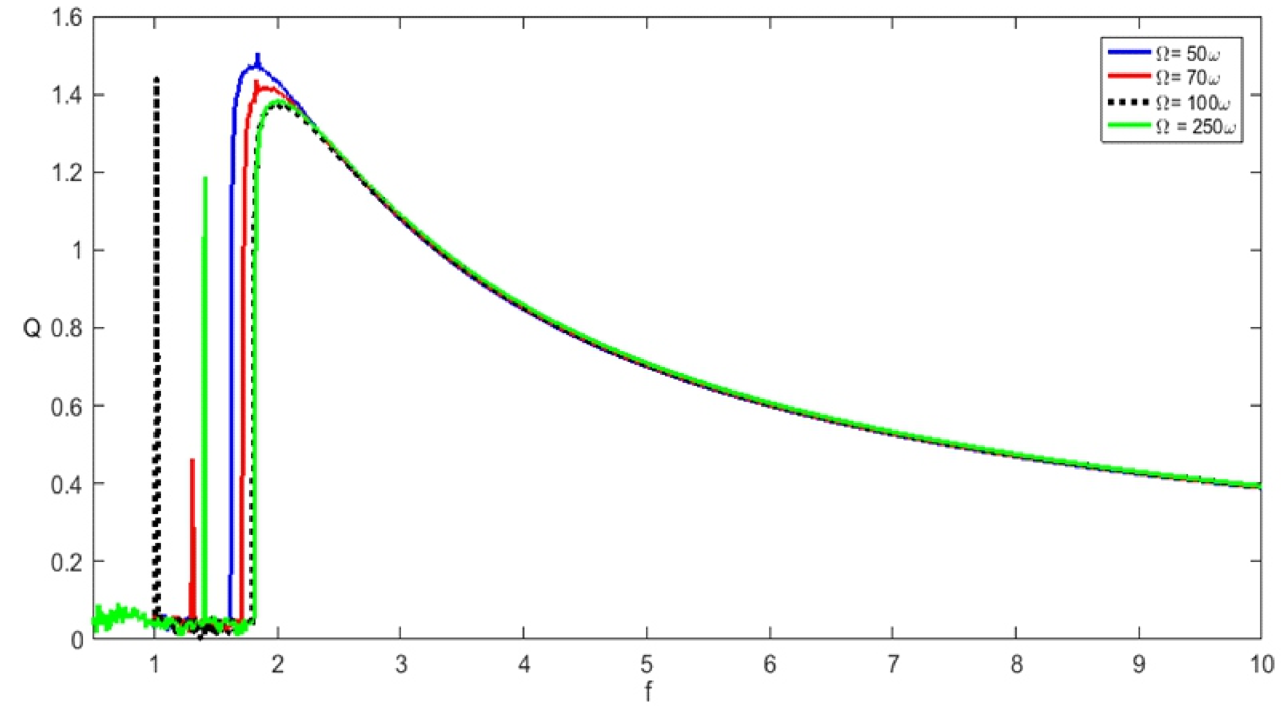

Figure 7: Response plot of Q against f, for different values of Ω=50ω,70ω, 100ω and 250ω, with other parameters fixed, $f = 0.08$ $g = 20$, $\omega = 1$ and $a = 0.3$.

Next, we also illustrate the mechanism of vibrational resonance in figure 5 with the superimposed response plot of Q against Ω, for different values of the amplitude of the fast periodic force (g=*10*,*30* and *50*), and other parameters of the system fixed. It is observed from the figures of the superimposed response curves (Figure 3,4 and 5), that the maximum response amplitude doesn't exceed the maximum response observed for $a = 0.3$, ($Q \leq 35.26$). This is in agreement with the report of Roy-Layinde *et al.* (2016); that the effective nonlinear dissipation of a system plays a crucial contributory role in the occurrence of VR. To further numerically investigate the occurrence of VR in a Chua's system, a superimposed response plot of Q against g, for different values of the amplitude of the low frequency driven force is plotted in figure 6 with other parameters of the system fixed at $\omega = 1$, $\Omega = 10\omega$,and $a = 0.3$.The system is greatly enhanced at the lower value of the amplitude of the slow driving force with maximum response amplitude ($Q = 47.02$). A F further increase in the amplitude of the slow periodic force reduces the response. This negates the earlier report of Jothimurugan *et al.* (2013) on experimental investigation of VR in a Chua's circuit; Q_{max} increases with an increase in f. This may be traced to the fact that the linear, piece-wise function in the Chua's circuit is replaced with a cubic nonlinear function. It is worth noting that the response curves respond at the same point ($g = 5$) and also peaks at the same point ($g = 8$). On this note, Figure 7 shows that resonance can be observed within certain parameter regime of the amplitude of the low frequency driven force, such that the system's response can only be modulated within this regime before saturated, (further modulation at $\Omega = 100$ω).

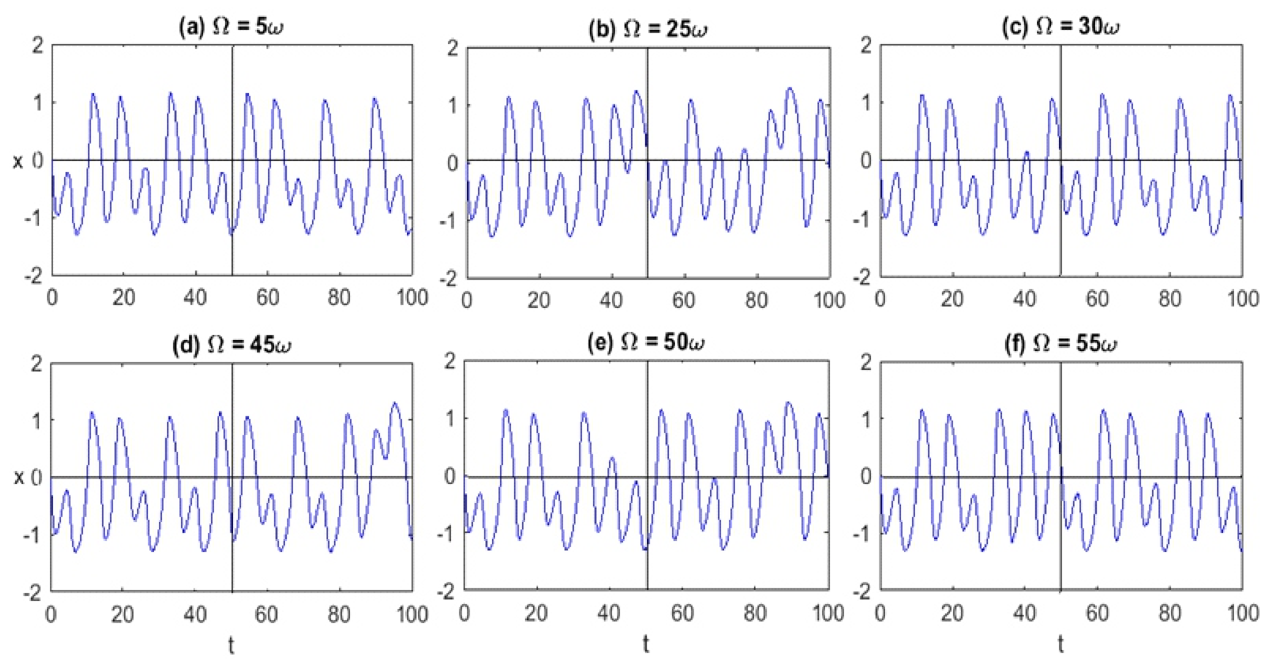

Figure 2(a)-(f): Time evolution of the attractors for different high frequency. The corresponding parameters are the same as those in figure 1.

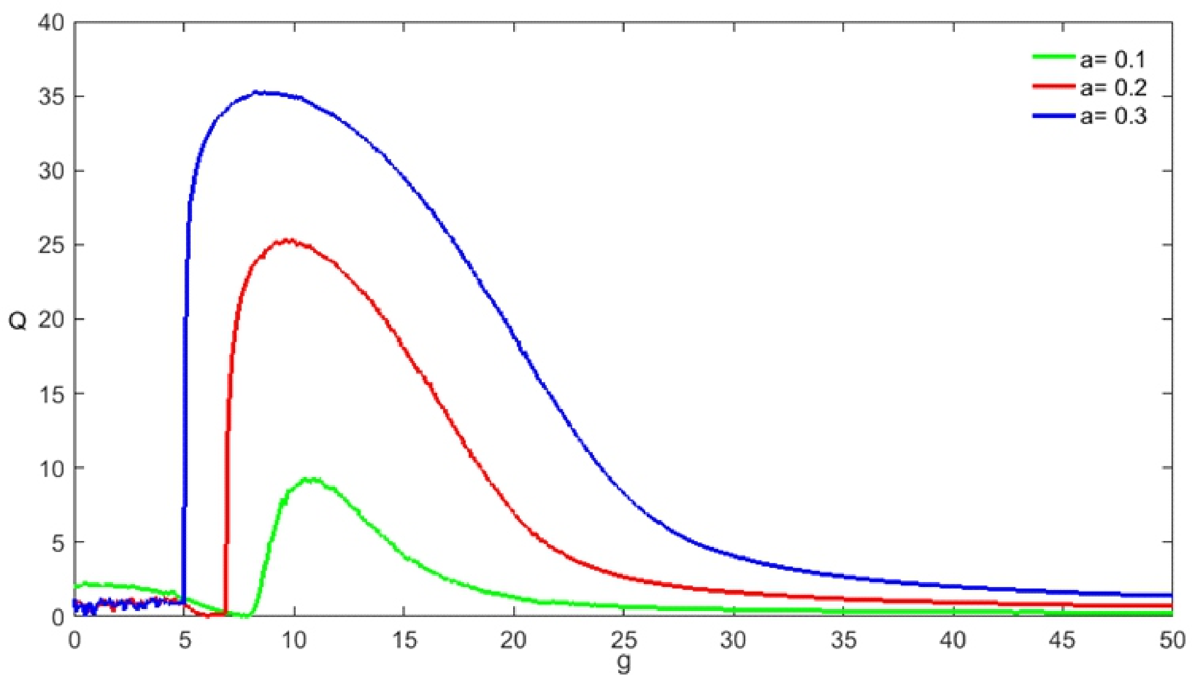

Figure 3:Superimposed response plot of Q against g, for different values of a =0.1, 0.2 and 0.3, with other parameters fixed at f = 0.08, ω = 1 and Ω = 10ω.

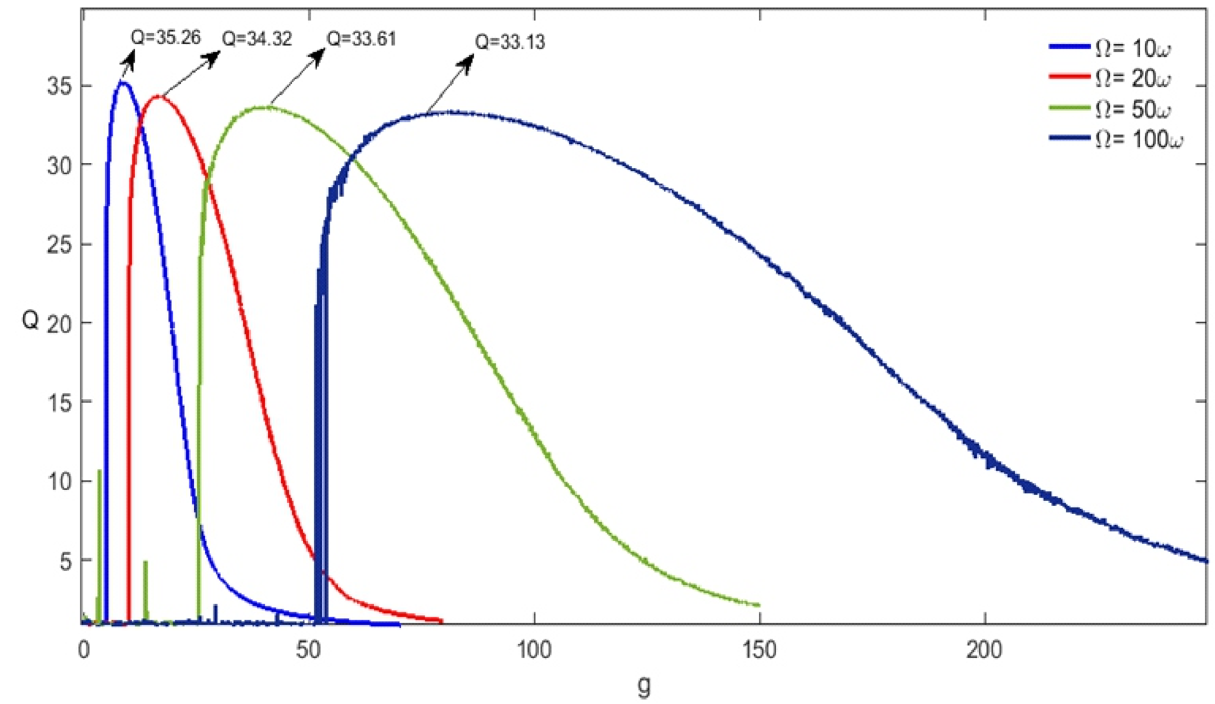

Figure 4:Superimposed response plot of Q against g, for different values of Ω=10ω,20ω, 50ω and 100ω, with other parameters fixed at f = 0.08, ω =1 and a = 0.3.

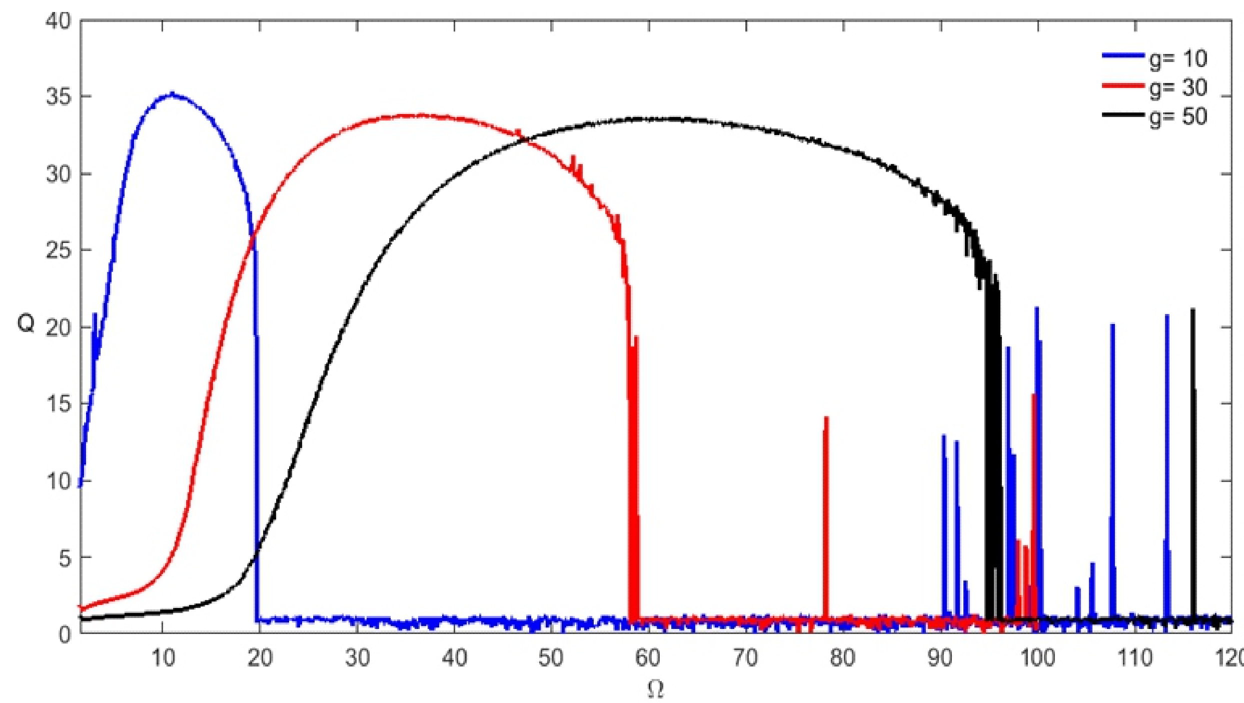

Figure 5: Superimposed response plot of Q against Ω, for different values of g =10,30 and 50, with other parameters fixed, f = 0.08, ω =1 and a = 0.3.

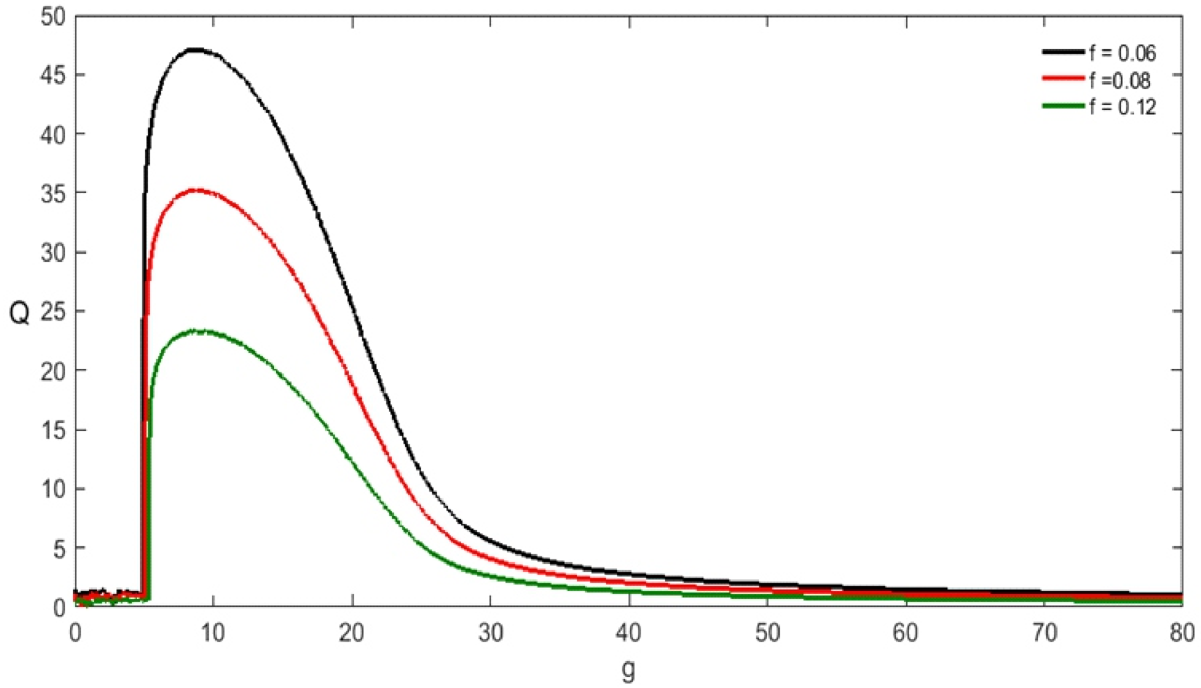

Figure 6: Superimposed response plot of Q against g, for different values of f = 0.06, 0.08 and 0.12, with other parameters fixed, Ω= 10ω, ω =1 and a = 0.3.

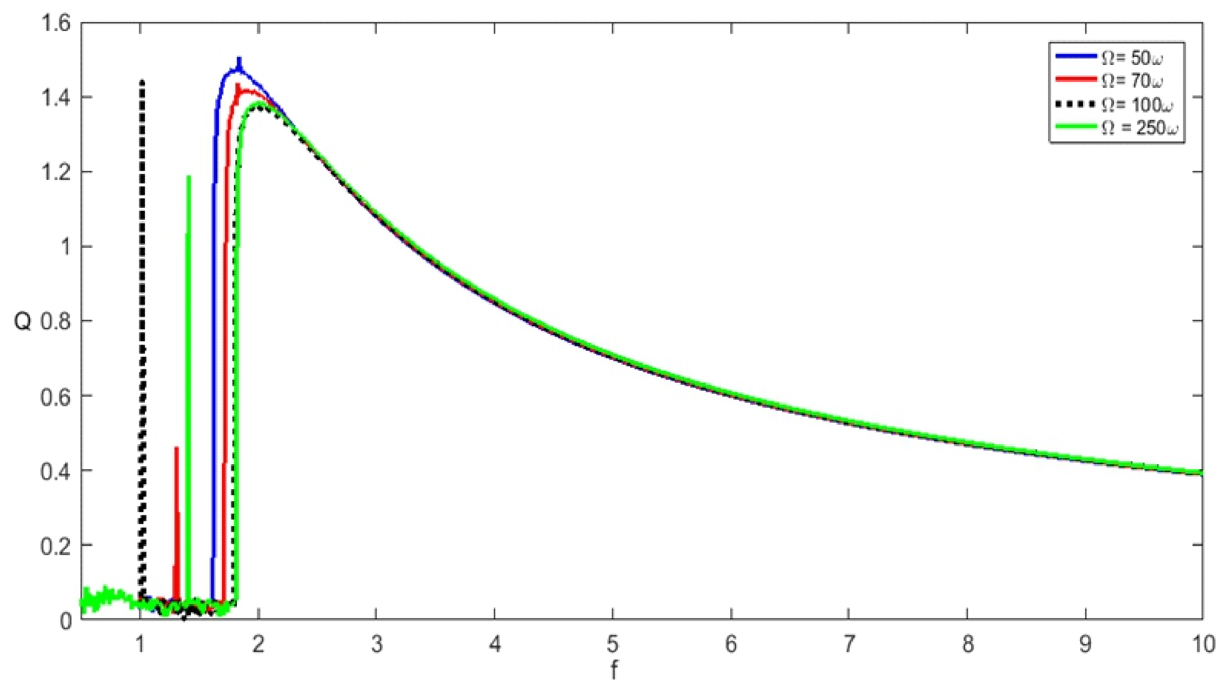

Figure 7: Response plot of Q against f, for different values of Ω=50ω,70ω, 100ω and 250ω, with other parameters fixed, f = 0.08,g = 20, ω = 1 and a = 0.3.

5.0 CONCLUSION

This research rigorously reports a numerical investigation of the occurrence of VR in a modified Chua's oscillator with cubic nonlinearity. Here the appearance and disappearance of the two periodic attractors of the Chua's circuit can be controlled by the frequency of the fast periodic force. The Chua's oscillator considered respond greatly and showed a very high sensitive dependence on the coefficient of dissipation, different from the traditional VR theory; modulation could be controlled via the coefficient of dissipation in a Chua's circuit. Our report has a potential application in electronic communication or in telecommunication where the quality of a weak low frequency signal at the input section needs to be greatly improved and well filtered before getting to the out put terminals.

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