ON THE RESPONSE OF VIBRATION ANALYSIS OF BEAM SUBJECTED TO MOVING FORCE AND MOVING MASS

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ABSTRACT

In this paper, vibration of beam subjected to moving force and moving mass is considered. Finite Fourier Sine transform with method of undetermined coefficient is used to solve the governing partial differential equation of order four. It was found that the response amplitude increases as the mass of the load increases for the case of moving mass while the response amplitude for the case of moving mass is not affected by increase in mass of the load. Also analysis shows that the response amplitude for the case of moving force is greater than that of moving mass.

KEYWORDS: Beam, Moving Mass, Moving Force, Finite Difference, Load, Mass, Concentrated & Vibration

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INTRODUCTION

Vibration is a mechanical phenomenon whereby oscillations (periodic or random) occur about an equilibrium point. It is the motion of a particle or a body of system of connected bodies displaced from a place of equilibrium (Abu, 2003; Akinpelu, 2012; Awodola, 2005; Chen & Jien, 2001). Beam is a structural element that is capable of withstanding load primarily by resisting bending. Beams are commonly found in many types of structures. Beams are characterized by their profile (shape of their cross section), their strength and material. Beams are typically made of steel, reinforced concrete or wood. They are also described by how they are supported, examples are simply supported beam, cantilever beam, overhanging beam and so on (Usman & Muibi, 2019). Beams deflect when loaded and this must be limited to avoid damage and distress. For instance, large deflection in a steel beam supporting a partition could cause unacceptable cracking in the plaster (Gbadeyan & Idowu, 2002; Gbadeyan & Usman, 2003; Idowu *et al.,* 2004; Ismail, 2011).

The purpose of vibrational analysis is to know the structural behavior under the influence of various loads and to get the necessary information such as deformation, moments and dynamic forces (Kozien, 2013; Naguleswan, 2006).

A Simply-supported beam has a hinged connection at one end and roller connection at the other end. Vibration analysis of a Simply-supportedbeam is essential as it helps us to examine a number of real life systems. For example, in the automotive industry, the leaf spring suspension system study can be supposed to a Simply-supported beam analysis (Usman & Adeboye, 2019). The numerical theory on Simply-supportedbeam has many applications for the calculations related to long span railway and highway bridges. The moving load may be considered as either a moving force (without inertia effect) or a moving mass (having inertia effect). The vibration analysis of beam structure with moving load is a fundamental problem in structural dynamics. The moving load varies in position as well as time. The problem of vibration of beam subjected to a moving load has been studied by many researchers (Usman, 2003).

FORMULATION OF THE PROBLEM

Consider a Simply-supported beam of finite length

L subjected to a moving load, the differential equation for the deflection of the Simply-supported beam subjected to a moving load when the beam is of constant flexural rigidity EI is given (Gbadeyan & Usman, 2006).

$$EIW^{(iv)}(x,t)+\rho AW_{tt}(x,t)=F(x,t) \qquad (1)$$

E is the modulus of elasticity
I is the constant moment of inertial of the beam's
A cross section about the axis.
W(x,t) is the deflection of the beam
ρ is the density of the mass
t is the mass
x is the spatial coordinate and
P(x,t) is the displaced force (i.e. the resultant concentrated force caused by moving mass).
The displaced force per unit length *P(x,t)* is defined as

$$P(x,t)=F\delta(x-vt) \qquad (2)$$

Where *δ(x-vt)* is the dirac-delta function defined as

$$\delta(x-vt)=\begin{cases}0 & \text{for } x=0\\ \infty & \text{for } x\neq 0\end{cases}$$

With the property that

$$\int_a^b \delta(x-vt)f(x)=\begin{cases}f(v,t) & a<vt<b\\ 0 & a<b<vt\\ 0 & vt<a<b\end{cases} \qquad (3)$$

Substituting equation (2) into equation (1) we have

$$EIW^{(iv)}(x,t)+\rho AW_{tt}(x,t)=F\delta(x-Vt) \qquad (4)$$

For $0<vt<L, 0<x<L, F$
is the force due to gravity and V is the velocity of the load. The above governing equation is subjected to the following boundary conditions.

$$w(0,t)=w(L,t)=0 \qquad (5)$$

$$W_{xx}(0,t)=W_{xx}(L,t)=0 \qquad (6)$$

Finally, the initial conditions are:

$$w(\bar{x},0)=w_0(x)=0 \qquad (7)$$

$$W_t(x,0)=W_0(x)=0 \qquad (8)$$

Method of Solution

We proceed to solve the above initial-boundary value problem described by

$$W(x,t)=X(x)T(t) \qquad (9)$$

Where X is the spatial function and T is the time function. To find the solution, finite Fourier sine transform method is adapted given by:

$$f_n(t)=\int_0^L F(x,t)\sin\frac{n\pi}{L}x\,dx \qquad (10)$$

With the inverse

$$\bar{F}(x,t)=\frac{2}{L}\sum_{n=1}^{\infty} f_n(t)\sin\frac{n\pi}{L}x \qquad (11)$$

We will now consider the solution for moving force and moving mass

Moving Force Problem

If the inertial effect of mass is neglected, then the reacting force *M* is given by Newton's second law which is

$$F=-Mg \qquad (12)$$

$$W(x,t)=\sum_{n=1}^{\infty} W_n(x,t)-\sum_{n=1}^{\infty} T_n(t)\sin\frac{n\pi}{L}x \qquad (13)$$

Find the function $f_n(t)$, where

$$F(x,t)=Mg\delta(x-vt) \qquad (14)$$

So we have

$$f_n(t)=-2g\frac{M}{L}\int_0^L\left[\delta(x-vt)\sin\frac{n\pi}{L}x\right]dx \qquad (15)$$

Applying the property of Dirac-delta function given in (3), we have

$$f_n(t)=-2g\frac{M}{L}\sin\frac{n\pi v}{L}t \qquad (16)$$

Substituting (15) into (11) after carrying out integration by using integration by part

$$F(x,t)=-2g\frac{M}{L}\sum_{n=1}^{\infty}\sin\frac{n\pi v}{L}t\sin\frac{n\pi}{L}x \qquad (17)$$

Substituting (17) into (1) above, we have

$$EI\left(\frac{n\pi}{L}\right)^4\sum_{n=1}^{\infty}T_n(t)\sin\frac{n\pi}{L}x+\rho A\sum_{n=1}^{\infty}T_n''(t)\sin\frac{n\pi}{L}x=-=-2g\frac{M}{L}\sum_{n=1}^{\infty}\sin\frac{n\pi v}{L}t\sin\frac{n\pi}{L}x \qquad (18)$$

Divide through by

$$\rho A\sum_{n=1}^{\infty}\sin\frac{n\pi}{L} \qquad (19a)$$

We have

$$T_n''(t)+\omega_n^2T_n(k)=k\sin\bar{\omega}_n(t) \qquad (19b)$$

Where

$$\omega_n^2=\frac{KI}{\rho A}\left(\frac{n\pi}{L}\right)^4,\ \bar{\omega}_n(t)=\frac{n\pi v}{L}\quad k=-\frac{2Mg}{\rho AL}$$

Solving the homogeneous part, we have the solution of the form

$$T_A(t)=G_n\sin\omega_n(t)+H_n\cos\omega_n(t) \qquad (20)$$

Using the method of undetermined coefficient, let

$$T_p(t)=A_n\sin\bar{\omega}_n(t)+B_n\cos\bar{\omega}_n(t) \qquad (21)$$

Differentiating (21) and inserting back into (20), we have

$$-\bar{\omega}_n(t)A_n-\bar{\omega}_n^2(t)B_n\cos\bar{\omega}_n(t)+\bar{\omega}_n^2(t)A_n\sin\bar{\omega}_n(t)+\bar{\omega}_n^2B_n\cos\bar{\omega}_n(t) \qquad (22)$$

$$\sin\bar{\omega}_n(t):\quad -\bar{\omega}_nA_n+\bar{\omega}_n^2A_n=k \qquad (23)$$

$$\cos\bar{\omega}_n(t):\quad -\bar{\omega}_nB_n+\bar{\omega}_n^2B_n=0 \qquad (24)$$

Solve equations (23) and (24) simultaneously when $B_n=0$, from (23), we have that

$$A_n=\frac{k}{(\omega_n^2-\bar{\omega}_n^2)} \qquad (25)$$

Hence

$$T_p(t)=\frac{k}{(\omega_n^2-\bar{\omega}_n^2)}\sin\bar{\omega}_n(t) \qquad (26)$$

So, from (21) and (26), we have

$$T_n(t)=G_n\sin\omega_n(t)+H_n\cos\omega_n(t)+\frac{k}{(\omega_n^2-\bar{\omega}_n^2)}\sin\bar{\omega}_n(t) \qquad (27)$$

Substituting into the initial conditions as given in (8) and (9), so we have

$$\sum_{n=1}^{\infty}T_n(0)\sin\frac{n\pi}{L}x=0 \qquad (28)$$

and

$$\sum_{n=1}^{\infty}T_n'(0)\sin\frac{n\pi}{L}x'=0 \qquad (29)$$

$$T_n(t) = G_n \sin \omega_n\ (t) + H_n \cos \omega_n\ (t) + \frac{k}{(\omega_n^2 - \bar{\omega}_n^2)} \sin \bar{\omega}_n\ (t) \quad (30)$$

$$T_n'(t) = G_n \cos \omega_n\ (t) - H_n \sin \omega_n\ (t) + \frac{k}{(\omega_n^2 - \bar{\omega}_n^2)} \bar{\omega}_n \cos \bar{\omega}_n\ (t) \quad (31)$$

Substituting the expression for $T_n(t)$ and $T'_n(t)$ in the initial conditions, we have $H_n = 90$ (32)

$$G_n = \frac{-k}{\bar{\omega}_n(\omega_n^2 - \bar{\omega}_n^2)} \bar{\omega}_n \quad (33)$$

So (27) becomes $T_n(t) = \frac{k}{(\omega_n^2 - \bar{\omega}_n^2)} [\sin \omega_n\ (t) - R_n \sin \bar{\omega}_n(t)]$ (34)

where $R_n = \frac{\bar{\omega}_n}{\omega_n}$

Substituting (34) into (13), we have

$$W(x,t) = \sum_{n=1}^{\infty} \frac{k}{(\omega_n^2 - \bar{\omega}_n^2)} [\sin \omega_n\ (t) - R_n \sin \bar{\omega}_n(t)] \quad (35)$$

Equation (35) is the deflection of beam subjected to moving force

Moving Mass Problem

Consider the inertial effect of moving mass, the reaction force F displaced by mass M is given by Newton's second law which is

$$F = -Mg - M \frac{d^2W(x,t)|_{vt}}{dt^2} \quad (36)$$

Here, $\frac{d^2W(x,t)|_{vt}}{dt^2}$ is the transverse displacement of the load and g is the acceleration due to gravity.

Substituting (36) into (2), we have

$$P(x,t) = M\left(-g - \frac{d^2W(x,t)|_{vt}}{dt^2}\right)\delta(x - vt) \quad (37)$$

Substituting (37) into (1), the equation becomes

$$EIW^{(iv)}(\lambda)(t) + \rho AW_{tt}(\lambda) = M\left(-g - \frac{d^2W(x,t)|_{vt}}{dt^2}\right)\delta(x - vt) \quad (38)$$

Where $W_{tt}(x,t) = W_{xt}(x,t) + 2VW_{xx}(x,t) + V^2W_{xt}(x,t)$ (39)

Substituting (39) into (38), we have

$$EIX_n^{(iv)}(x)T_n(t) + \rho AX_n(x)T_n''(t) = -Mg\delta(x - vt) \quad (40)$$

Applying the Finite Fourier Sine Transform in (39) and (40) Where

$$\bar{F}(x,t) = -M\left[g - \left(X_n(x)T_n''(t) + 2vX_n'(x)T_n'(t) + v^2X_n''(x)T_n(t)\right)\right]\delta(x - vt) \quad (41)$$

We have

$$f_n(t) = -2g\int_0^L X_n(x)\delta x - 2\frac{M}{L}X_n(vt)\int_0^L \begin{pmatrix} X_n(x)T_n''(t) + 2vX_n(x)T_n'(t) \\ +v^2X_n''(x)T_{n(t)} \end{pmatrix}\delta(x - vt) \quad (42)$$

Substituting (41) and (42) into (11), we have

$$\bar{F}(x,t) = \Big[-2g\int_0^L X_n(x)\delta x - 2\frac{M}{L}X_n(vt)\int_0^L \big(X_n(x)T_n''(t) + 2vX_n'(x)T_n'(t) + v^2X_n''(x)T_n(t)\big)\delta(x - vt)\Big]\sin\frac{n\pi}{L} \quad (43)$$

Since $X_n(x) = \sin\frac{n\pi}{L}x$ we then have

$$X_n(vt) = \sin\frac{n\pi v}{L}x \quad (44)$$

Substituting (43) and (44) into (40), we have

$$EI\left(\frac{n\pi}{L}\right)^4 \sum_{n=1}^{\infty} T_n(t)\sin\frac{n\pi}{L}x + \rho A\sum_{n=1}^{\infty} T_n''(t)\sin\frac{n\pi v}{L}$$

$$= \sum_{n=1}^{\infty}\left[-2g\frac{M}{L}\sin\frac{n\pi v}{L}t - 2\frac{M}{L}\begin{pmatrix} \sin\frac{n\pi v}{L}tT_n'(t) + 2v\frac{n\pi}{L}\cos\frac{n\pi v}{L}t\sin\frac{n\pi v}{L}tT_n''(t) \\ -\left(\frac{n\pi}{L}\right)^2 v^2\sin\frac{n\pi v}{L}t\sin\frac{n\pi v}{L}t\sin\frac{n\pi v}{L}tT_n(t) \end{pmatrix}\right]\sin\frac{n\pi}{L}x \quad (45)$$

Divide both sides by $\sum_{n=1}^{\infty}\sin\frac{n\pi}{L}x$

We then have $T_n''(t) + Q_1T'(t) + Q_2T_n(t) = Q_3\sin\bar{\omega}_n(t)$ (46)

Where

$$Q_1 = \frac{H_5}{H_2 + H_4},\quad Q_2 = \frac{H_1 - H_6}{H_2 + H_4},\quad Q_3 = \frac{H_3}{H_2 + H_4},\quad H_1 = EI\left(\frac{n\pi}{L}\right)^4,\quad H_2 = \rho A,\quad H_3 = -2g\frac{M}{L},$$

$$\bar{\omega}_n = \frac{n\pi v}{L},\quad H_4 = 2\frac{M}{L}\sin\frac{n\pi v}{L}t\sin\frac{n\pi v}{L}t,\quad H_5 = 4Mv\frac{n\pi}{L^2}\cos\frac{n\pi v}{L}t\sin\frac{n\pi v}{L}t, \quad (47)$$

Solving the homogeneous part of (46), we obtained solution of the form

$$m_{1,2} = -\frac{Q_1}{2} \pm \sqrt{\left(\frac{Q_1}{2}\right)^2 - Q_2}$$

Using the method of undetermined coefficient, let

$$T_p(t) = F_n \sin\bar{\omega}_n t + G_n \cos\bar{\omega}_n t \quad (48)$$

Differentiating (48), we have

$$G_n = \frac{Q_3 \omega_n Q_1}{(\bar{\omega}_n^2 - Q_2)(Q_2 - \bar{\omega}_n^2) - \bar{\omega}_n^2 Q_1^2} \quad (49)$$

and

$$F_n = \frac{Q_3 \omega_n Q_1(\bar{\omega}_n^2 - Q_2)}{\omega_n Q_1(\bar{\omega}_n^2 - Q_2)(Q_2 - \bar{\omega}_n^2) - \bar{\omega}_n^3 Q_1^3} \quad (50)$$

So that

$$T_p(t) = \frac{Q_3 \omega_n Q_1(\bar{\omega}_n^2 - Q_2)}{\omega_n Q_1(\bar{\omega}_n^2 - Q_2)(Q_2 - \bar{\omega}_n^2) - \bar{\omega}_n^3 Q_1^3} \sin\bar{\omega}_n t + \frac{Q_3 \omega_n Q_1}{(\bar{\omega}_n^2 - Q_2)(Q_2 - \bar{\omega}_n^2) - \bar{\omega}_n^2 Q_1^2} \quad (51)$$

Hence, we have

$$T_n(t) = Ae^{m_1 t} + Be^{m_2 t} + \bar{R}_n(K_n \sin\bar{\omega}_n t + \cos\bar{\omega}_n t) \quad (52)$$

Where

$$R_n = \frac{Q_3 \omega_n Q_1(\bar{\omega}_n^2 - Q_2)}{\omega_n Q_1(\bar{\omega}_n^2 - Q_2)(Q_2 - \bar{\omega}_n^2) - \bar{\omega}_n^3 Q_1^3}, \quad K_n = \frac{(\bar{\omega}_n^2 - Q_2)}{\omega_n Q_1} \quad (53)$$

Substituting the expression for $T_n(t)$ and $T'_n(t)$ and then solving it, we get

$$\sum_{n=1}^{\infty} T_n(0) \sin\frac{n\pi}{L} x = 0 \quad (54)$$

and

$$\sum_{n=1}^{\infty} T'_n(0) \sin\frac{n\pi}{L} x = 0$$

$$T_n(t) = A_n e^{m_1 t} + B_n e^{m_2 t} + \bar{R}_n(K_n \sin\bar{\omega}_n t + \cos\bar{\omega}_n t) \quad (55)$$

$$T'_n(t) = T_n(t) = A_n e^{m_1 t} + B_n e^{m_2 t} + \bar{R}_n(K_n \sin\bar{\omega}_n t + \cos\bar{\omega}_n t) \quad (56)$$

Substituting the expression $T_n(t)$ and $T'_n(t)$ and then solving it, we get

$$B_n = \frac{R_n(m_1 - K_n)}{(m_1 - m_2)} \quad (57)$$

and

$$A_n = \frac{R_n(K_n - m_1)}{(m_1 - m_2)} \quad (58)$$

$$T_n(t) - \frac{\bar{R}_n}{(m_1 - m_2)}[(K_n - m_2)e^{m_1 t} + (m_1 - K_n)e^{m_2 t} + \bar{R}_n(K_n \sin\bar{\omega}_n t + \cos\bar{\omega}_n t)] \quad (59)$$

$$W(x,t) = \sum_{n=1}^{\infty}\left\{\frac{\bar{R}_n}{(m_1 - m_2)}[(K_n - m_2)e^{m_1 t} + (m_1 - K_n)e^{m_2 t} + \bar{R}_n(K_n \sin\bar{\omega}_n t + \cos\bar{\omega}_n t)]\right\}\sin\frac{n\pi}{L}x \quad (60)$$

Equation (60) is the deflection of the beam subjected to moving mass.

RESULTS

MATLAB was used and thenumerical data,given below, were used

$\rho A = 0.98kgm^{-1}$, $E = 2.10 \times 10^{11}N$, $b = 0.001m$, $d = 0.0001m$, $M = 50{,}100kg$,

$L = 6m$, $m = 7kg$, $v = 28m/s$

Figures 1-4 show the deflection profiles of the beam depicted graphically to demonstrate the effect of the velocity of the load, time at which the load moves and increase in mass of the load.

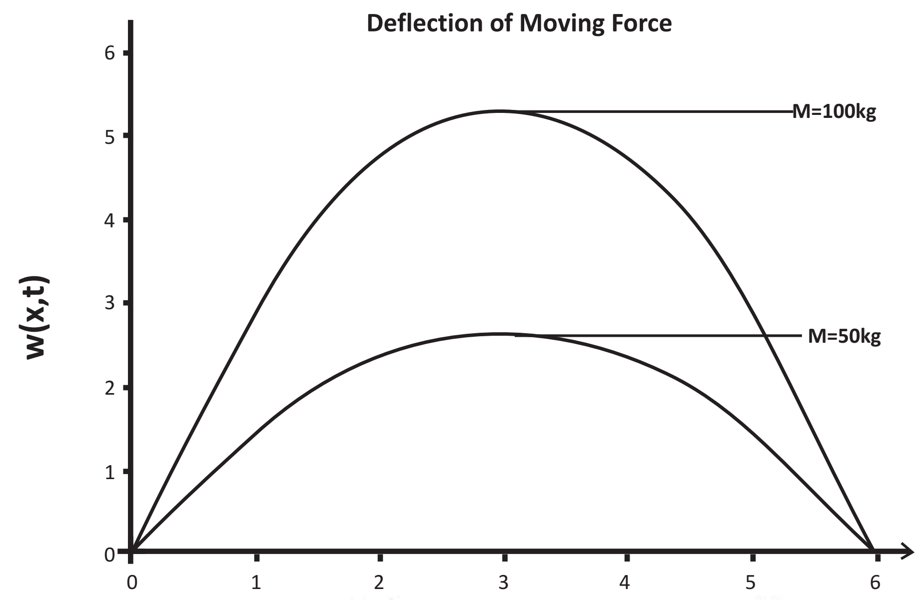

Figure 1: Deflection of beam subjected to moving force

x

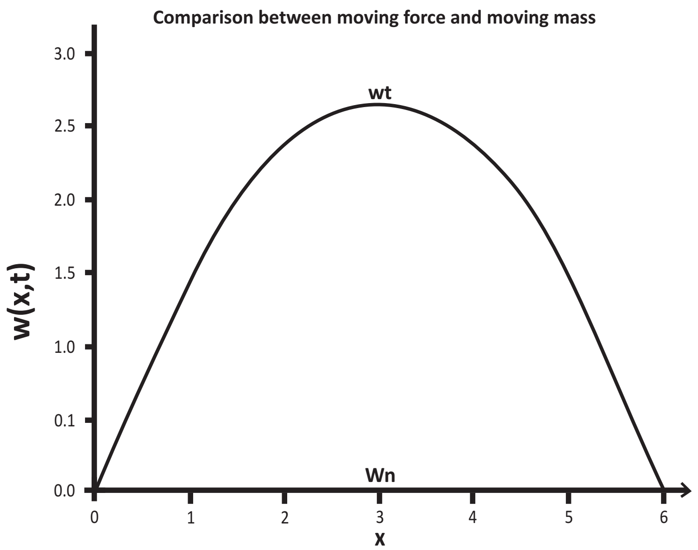

Figure 3: Comparison between moving force and moving mass for M = 59kg.

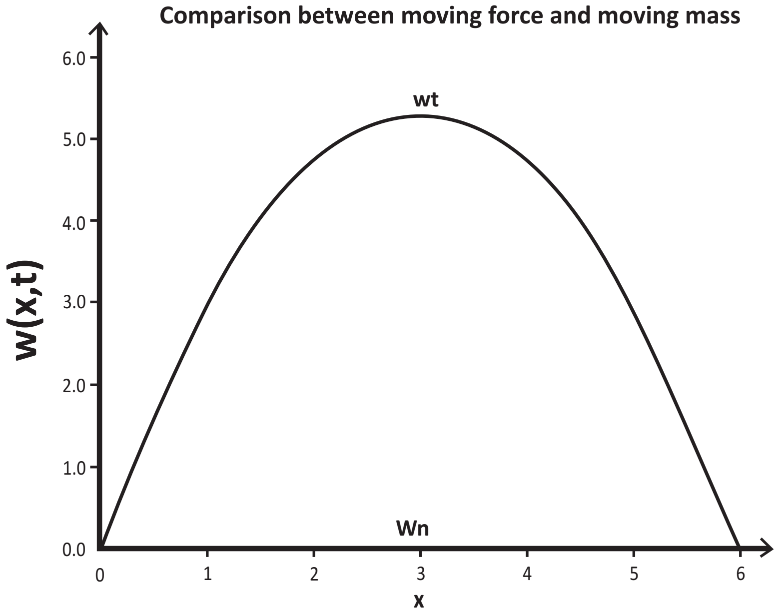

Figure 4: Comparison between moving force and moving mass for M=100kg.

DISCUSSION AND CONCLUSION

The response of vibration analysis of beam subjected to moving force and moving mass was investigated. Finite Fourier Sine transform with undetermined coefficient was used to solve the governing equation.

From the response profile of the beam, it was found that the response amplitude increases as the mass of the load increases for the case of moving force while the response amplitude for the case of moving mass is not affected by increase in mass of the load. Also analysis shows that the response amplitude for the case of moving force is greater than that of moving mass.

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