A Univariate Time Series Analysis of Nigeria’s Monthly Inflation Rate

Olatayo, Timothy O. & Taiwo, Abass I.
Department of Mathematical Sciences, Olabisi Onabanjo University, Ago-Iwoye
bisi.olatayo@oouagoiwoye.edu.ng & taiwo.abass@oouagoiwoye.edu.ng

ABSTRACT
Inflation refers to general increase in prices and fall in the purchasing power or value of money. It has always been undesirable for any nation’s economy in the world. This paper discusses and analyses the fluctuations and volatility in Nigeria's inflation rates. The methodology employed in the analysis and modelling of the inflation rates was Seasonal Autoregressive Integrated Moving Average (SARIMA) model. The identification stage of the model building suggests four models for Nigeria’s inflation forecast, but was validated in the estimation stage using Akaike Information Criteria (AIC), Schwart Bayesian Criteria (SBC), Hanna Quinn Criteria (HQC) parameters. The model was diagnosed and the results showed that the model was adequate and parsimonious. Hence, \( \hat{\phi} = 1.08X_{t-2} - 0.08X_{t-1} + \theta_{t-1} - 0.978X_{t-12} - 0.238X_{t-24} + 0.271X_{t-36} \) was fitted to obtain forecast values on monthly bases with minimum root mean square error (RMSE), mean absolute error (MAE) and mean absolute percentage error (MAPE) parameters respectively. In conclusion, the model was a good fit with minimum parameters and the forecast generated revealed high fluctuation and volatility in Nigeria’s inflation rates. The forecast results will help policy makers to gain an insight into more appropriate economic and monetary policy in order to combat the predicted rise and fall of Nigerian inflation rates.

Keywords: Inflation rate, Seasonal Autoregressive Integrated Moving Average, Forecasting, Parameters and Model Building.

INTRODUCTION
Inflation is the rate at which the general financial level of prices for goods and services is rising and, consequently, the purchasing power of currency is falling. As inflation rises, your money you buys or pays for a smaller percentage of a good or a service. Central banks attempt to reduce inflation and avoid deflation, in order to keep the economy moving smoothly.

Considering the detrimental impacts of inflation on the economy, a consensus is developed among the worlds’ leading central banks that the chief goal of monetary policy is to control inflation in the economy since high inflation and high interest rates hamper economic growth by discouraging investment and reducing productivity of the country. In most of the countries, normal interest rate is considered as the targeting instrument for achieving inflation target. Hence, inflation is a problem in all facets of life and in all economic entities.

The government of any nation is saddled with the responsibility of ensuring that her plans and programmes are not frustrated by unpredictable and galloping prices. Every firm desires a stable macro-economic environment that is devoid of unrepentant price change that can negatively affect reliable forecasting and planning. An individual also strives that he is not left worse off by unexpected price increase. All these bring home the need to explore the study of inflation on monthly basis so as to form a timeless and dependable model of its tendencies (Taiwo, 2011).

(Sloman et al., 2007) explain that inflation may either be demand pull inflation or cost push inflation. Demand pull inflation is caused by persistent rise in aggregate demand thus firms respond by raising prices and partly by increasing output. Cost push inflation is associated with persistent increase in the costs experienced by firms. Firms respond by raising prices and passing the costs on to the consumers and by partly cutting back on production. (Hendry, 2006) agrees that inflation is the resultant of excess demands and supplies in the economy.

(Tucker, 2007) observed that there are many measures of inflation, because there are many different price indices relating to different sectors of the economy. Two widely known indices for which inflation rates are reported in many countries are the Consumer Price Index (CPI) which measures prices that affect typical consumers, and the Gross Domestic Product (GDP) deflator which measures prices of locally-produced goods and services.
Price stability is one of the main objectives of every government. It is an important economic indicator that governments, politicians, economists and other stakeholders use as the basis of argument when debating on the state of the economy (Suleman et al., 2012). In recent years, rising inflation has become one of the major economic challenges facing most countries in the world especially, developing countries such as Nigeria. (David, 2001) described inflation as a major focus of economic policy worldwide. This is rightly said since inflation is the frequently used economic indicator of the performance of a country’s economy.

Having understood this, it is necessary to study the structure and the pattern of Nigeria’s monthly inflation rates and as well give recommendations to policy makers that inflation should be specially cared for among the macro economic variables. This will assist central banks in their attempt to limit inflation, avoid deflation and keep the economy moving smoothly. We proposed a model by using seasonal Box Jenkins’ model. The model was used to obtain future values for inflation to assist the economic and monetary policy makers in Nigeria.

Materials and Methods
Here we will discuss several modifications made to the ARIMA model to account for seasonal and non-stationary behaviour. Often, the dependence on the past tends to occur most strongly at multiples of some underlying seasonal lags. Seasonal ARIMA (SARIMA) is used when the time series exhibits a seasonal variation. Natural phenomena such as temperature and rainfall, have strong components corresponding to seasons. Hence, the natural variability of many physical, biological, and economic processes tends to match with seasonal fluctuations. Because of this, it is appropriate to introduce autoregressive and moving average polynomials that identify with seasonal lags. The resulting pure seasonal autoregressive moving average model, say ARIMA(P, d, Q)s, then takes the form (Wei, 2006)

\[ \Phi_p(B^s)\varphi(B^{-s})x_t = \theta_q(B)w_t, \]  

with the following definition of the operators

\[ \Phi_p(B^s) = 1 - \phi_1B^s - \phi_2B^{2s} - \ldots - \phi_pB^{ps}, \]

are the seasonal autoregressive operator and the seasonal moving average operator of orders P and Q respectively, with seasonal period S. Analogous to the properties of non-seasonal ARMA models, the pure seasonal ARIMA(P, d, Q)s is causal only when the roots of \( \Phi_p(B^s) \) lie outside the unit circle, and it is invertible only when the roots of \( \theta_q(B^{-s}) \) lie outside the unit circle. In general, we can combine the seasonal and non-seasonal operators into a multiplicative seasonal autoregressive moving average model, denoted by ARIMA(p, q) × (P, Q)s and we can write the overall model as:

\[ \Phi_p(B^s)\varphi(B^{-s})x_t = \theta_q(B^{-s})\theta(B)w_t, \]  

A seasonal autoregressive notation (P) and a seasonal moving average notation (Q), will form the multiplicative seasonal autoregressive integrated moving average model, denoted by ARIMA(p, d, q)(P, D, Q)s, of (Box and Jenkins, 1976) and is given by:

\[ \Phi_p(B^s)\varphi(B^{-s}) = \alpha + \theta_q(B^{-s})\theta(B)w_t \]  

where \( w_t \) is the usual Gaussian white noise process. The ordinary autoregressive and moving average components are represented by polynomials \( \Phi(B) \) and \( \theta(B) \) of orders p and q respectively and the seasonal autoregressive and moving average components by \( \Phi_p(B^s) \) and \( \theta_q(B^{-s}) \) of orders P and Q respectively. The ordinary and seasonal difference components can be written as \( \nabla^d = (1 - B)^d \) and \( \nabla^s = (1 - B^s)^d \).

Model Identification
Autocorrelation function (ACF) and partial autocorrelation function (PACF) are the two most useful tools in time series model identification (Granger, 1986). The sample ACF measures the amount of linear dependence between observations in a time series that are separated by a lag k and auto-correlation of order is simply given as the correlation between \( x_k \) and \( x_{t-k} \) that is,

\[ \rho_k = \frac{E[(x_t - \bar{x})(x_{t-k} - \bar{x})]}{\sigma_x^2} \]  

While partial autocorrelation function can also be used for determining the possible order of seasonal autoregressive, non-seasonal autoregressive, moving average and seasonal moving average that should be incorporated in the model. To obtain an estimate for partial autocorrelations at , we can employ successive autoregressive estimation procedure. The first step is to model the series with finite autoregressive models of order given by (Box and Jenkins, 1976):

\[ x_t = \rho_1 + \sum_{i=1}^{\rho} \rho_{ik} x_{t-k} \]  

where \( \rho_{ik} \) is the autoregressive coefficient, and estimate of these coefficients using ordinary least squares or maximum likelihood estimation method gives the k-th sample partial autocorrelation (Hipel et al., 1977).

Parameter Estimation
After choosing the most appropriate model, we can use ordinary least squares or maximum likelihood estimation method to estimate the coefficients of the model. For the ordinary least squares method, we consider time series model given as

\[ x_t = \theta_1x_{t-1} + w_t, \quad t = 1, \ldots, n \]  

Then, the OLS estimator of \( \theta \) is given by

\[ \hat{\theta} = \frac{\sum_{i=1}^{n} x_{t-1}^2 - \sum_{i=1}^{n} x_{t-1}x_t}{\sum_{i=1}^{n} x_{t-1}^2 - \sum_{i=1}^{n} x_{t-1}^2} \]  

and for the maximum likelihood estimation, we use the unconditional log-likelihood function given as
where $S(\theta, \mu, \sigma^2)$ is the unconditional sum of square function given by

$$S(\theta, \mu, \sigma^2) = \sum_{x=\infty}^{n} \left[ \varepsilon(a_i/(\theta, \mu, \sigma, x)) \right]^2.$$  

### Diagnostic Checking

After fitting a provisional time series model, we can assess its adequacy using Ljung-Box $Q$ or $Q(r)$ statistic by check independence of residual. A test of hypothesis can be done for the model adequacy by choosing a level of significance and then comparing the value of the calculated $X^2$ with the $X^2$-table critical value. A useful test in these concepts is the portmanteau lack of fit test. This uses the entire residual sample with the null hypothesis that $H_0: \rho_1 = \rho_2 = \cdots = \rho_k = 0 \forall s \geq k$. The test statistic is calculated by the following equation (Davidson, 2000):

$$Q(r) = n'(n'-2) \sum_{j=1}^{k} \frac{\mu_j^2}{n'-j}$$

where $n' = (n - d)$, $n$ is the number of observations in the original time series, $\mu_j^2$ is the sample autocorrelation of the residuals at lag $j$ and $d$ is the degree of non-seasonal differencing used to transform the original time series values into stationary time series values and $k$ is a sufficiently large integer.

### Forecasting

There are two kinds of forecast and these are sample period forecast and post-sample period forecast. The former will be used to develop confidence in the model and the latter will be used to generate genuine desired forecasts. In forecasting, the goal is to predict future values of a time series, $x_{t+m}$, based on the data collected to the present, $x = \{x_1, x_2, \ldots, x_t\}$ (Olatayo and Alabi 2011), (Taiwo and Olatayo 2013). There are some measurements of the accuracy of forecasts and these are mean absolute error (MAE) defined as

$$MAE = \frac{1}{h+1} \sum_{t=0}^{h+1} (x_t - \hat{x}_t)^2$$

Root Mean Square forecast Error (RMSE) defined as

$$RMSE = \sqrt{\frac{1}{h+1} \sum_{t=0}^{h+1} (x_t - \hat{x}_t)^2}$$

and the mean absolute percentage error define as,

$$MAPE = \frac{100}{h+s} \sum_{t=0}^{h+s} \left| \frac{x_t - \hat{x}_t}{\hat{x}_t} \right|$$

where $t = s, 1, s, \ldots, \hat{h}, \hat{s}$ The actual and predicted values for corresponding $t$ values are denoted by $x_t$ and $\hat{x}_t$, respectively. The smaller the values of $RMSE$ and $MAPE$, the better the forecasting performance of the model (Olatayo & Taiwo 2012), (Olatayo & Adeboye 2013) (Olatayo et al., 2014).

### Result and Discussion

The data used was collected from Central Bank of Nigeria national Bureau of statistics from January 1990 to June 2014.

The time plot of Nigeria’s inflation rates is given in Figure 2. It exhibits an upward increasing trend and suggests that the given time series is non-stationary. The movement is secular in nature and is a small shift in the movement is expected.

The augmented dickey fuller and Phillips-Persons test show that Nigerian Inflation rate is stationary at the first difference that is $I_1$ at 1%, 5% and 10% level of significance with $p-value = 0.000$. Since the order of integration of the difference population series is one (1), then $d = 1$. (Rao 1994) and (Pfaff 2006).

### Model Building for Nigeria’s Monthly Inflation series

First, concentrating on the seasonal lags, the characteristics of the ACF and PACF in Figure 1 tend to show a strong peak at $h = 12$ in the autocorrelation function, combined with peaks at $h = 12$ and $24$ in the partial autocorrelation function. Hence, it appears that either: (i) the ACF is cutting off after lag 12 and the PACF is tailing off in the seasonal lags, (ii) the ACF and PACF are both tailing off in the seasonal lags. Table 1 suggests either (i) a seasonal moving average of order $Q = 1$, or (ii) due to the fact that both the ACF and PACF may be tailing off at the seasonal lags, perhaps both components, $P = 1$ and $Q = 0$, are needed. To identify between-season model, we focus the lags $h = 1, 2, \ldots, 24$ and identify order, based on Table 1. First, we set the ACF to be tailing-off and the PACF to be cut-off after lag 1, we identify $p = 1$ and $q = 1$. Also it is possible to think of the PACF to be tailing-off and the ACF to cut-off after lag 1, leading to identify that $P = 0$ and $Q = 1$.

Fitting the four models suggested by these observations, we obtain:

- $SARIMA (1,1,1) \times (1,0,1)_7$
- $SARIMA (1,1,1) \times (0,0,1)_7$
- $SARIMA (1,1,1) \times (1,0,0)_7$
- $SARIMA (0,1,1) \times (1,0,1)_12$
**Table 1. Correlogram**

<table>
<thead>
<tr>
<th>Autocorrelation</th>
<th>Partial Correlation</th>
<th>AC</th>
<th>PAC</th>
<th>Q- Stat</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.jpg" alt="Table image" /></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Where $X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \phi_3 X_{t-3} + \epsilon_t$ and for $SARIMA(0,1,1) \times (0,1,1)_d$ model, the fitted model is obtained as: $X_t = -0.074 + \epsilon_t$, $(1 - 0.621i)^2 \epsilon_t = (1 + 0.21i)(1 - 0.96i) \epsilon_t$, where $AIC = 3.34805$, $SIC = 3.43056$ and $HQCC = 3.54802$.

The model with the smaller information criteria is said to fit the data better. Since $SARIMA(1,1,1) \times (0,0,1)_d$ model has the lowest AIC and SBC, then this model is believed to estimate Nigeria's monthly inflation rate better than the other models.

The diagnosis checking was done using Breusch and Pagan (B-P) test for the homoscedasticity of the residuals and the results show that all calculated values are found to be smaller than the respective critical values, which indicate that the residual variance is constant. Therefore, the hypothesis that the residuals are white noise cannot be rejected indicating that the fitted model is adequate. That is, $SARIMA(1,1,1) \times (0,0,1)_d$, the model is adequate for modelling Nigeria's inflation rate. To test whether residuals from the fitted model come from normally distributed series, we use histogram and QQ-plot of the residual test and the results show that the residuals come from a normally distribution.

Since the model diagnostic tests show that all the parameter estimates are significant and the residual series is white noise, the estimation and diagnostic checking stages of the modelling process is complete. We can now proceed to forecasting the inflation series with $SARIMA(1,1,1) \times (0,0,1)_d$ fitted model. The $SARIMA(1,1,1) \times (0,0,1)_d$ model can be written as

$$(1 - \phi_1 B) \epsilon_t = (1 + \theta_1 B)(1 + \theta_2 B) \epsilon_t$$

The above equation can be re-expressed as:

$$X_{t+m} = \phi_1 X_{t+m-1} + \phi_2 X_{t+m-2} + \phi_3 X_{t+m-3} + \epsilon_t + \theta_1 \epsilon_{t+1} + \theta_2 \epsilon_{t+2} + \theta_3 \epsilon_{t+3}$$

In order to forecast one period ahead that is, $X_{t+1}$ equation (15) is increased by one unit throughout and it is as given by $X_{t+1} = (1 + \phi)X_t - \phi X_{t-1} - \phi_2 X_{t-2} - \phi_3 X_{t-3} + \epsilon_t + \theta_1 \epsilon_{t+1} + \theta_2 \epsilon_{t+2} + \theta_3 \epsilon_{t+3}$

The term $\epsilon_{t+1}$ is not known because the expected value of future random errors has been taken as zero. There are 138 data points from January 2004 to June 2015 used to build the SARIMA model and $\phi = 0.08, \theta_1 = 0.28, \theta_12 = 0.97$ and $\theta_12 = 0.271$.

Thus, (16) is given as

$$\hat{X}_{t+1} = 1.08X_{t+1} - 0.08X_{t+2} + \hat{\epsilon}_{t+1} - 0.97\hat{\epsilon}_{t+2} - 0.28\hat{\epsilon}_{t+3} + 0.371\hat{\epsilon}_{t+12}$$

In order to forecast inflation for the period 139 (that is, July 2015), equation (17) is given by

$$\hat{X}_{139} = 1.08X_{138} - 0.08X_{137} + \hat{\epsilon}_{139} - 0.97\hat{\epsilon}_{138} - 0.28\hat{\epsilon}_{139} + 0.271\hat{\epsilon}_{126}$$

$\hat{\epsilon}_{139} = 0$

$\hat{\epsilon}_{138} = \hat{\epsilon}_{137} = \hat{\epsilon}_{126} = 0$

The forecast quantity for period 139 can now be calculated as follows:
Once our model has been obtained and its parameters have been estimated, we can use it to make our predictions. Table 3 below summarizes 12 months up-front inflation forecast from July 2015 to June 2016.

**TABLE 2: Monthly Forecasting Inflation Values (July 2015 - June 2016).**

<table>
<thead>
<tr>
<th>Month(s)</th>
<th>Period(s)</th>
<th>Forecast %</th>
</tr>
</thead>
<tbody>
<tr>
<td>July</td>
<td>139</td>
<td>8.7</td>
</tr>
<tr>
<td>August</td>
<td>140</td>
<td>7.9</td>
</tr>
<tr>
<td>September</td>
<td>141</td>
<td>7.9</td>
</tr>
<tr>
<td>October</td>
<td>142</td>
<td>7.1</td>
</tr>
<tr>
<td>November</td>
<td>143</td>
<td>6.8</td>
</tr>
<tr>
<td>December</td>
<td>144</td>
<td>6.8</td>
</tr>
<tr>
<td>January</td>
<td>145</td>
<td>6.5</td>
</tr>
<tr>
<td>February</td>
<td>146</td>
<td>6.3</td>
</tr>
<tr>
<td>March</td>
<td>147</td>
<td>6.0</td>
</tr>
<tr>
<td>April</td>
<td>148</td>
<td>5.5</td>
</tr>
<tr>
<td>May</td>
<td>149</td>
<td>5.3</td>
</tr>
<tr>
<td>June</td>
<td>150</td>
<td>4.5</td>
</tr>
</tbody>
</table>

The result of the forecasting accuracy evaluation in Table 3 shows that the Root Mean Square Error is 0.052, Mean absolute error is 0.026, Mean Relative Percentage Error (MAPE) turns out to be 0.023 which is relatively less and Theil’s inequality coefficient (U-statistic) turns out to be 0.018, which is relatively close to zero. Besides this result, the bias and variance proportion are also very small which are 0.047 and 0.001 respectively. Thus, this measures indicate that the forecasting inaccuracy is low.

**Table 3: Forecasting Evaluation**

<table>
<thead>
<tr>
<th>Accuracy Measures</th>
<th>Monthly Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Root mean square error</td>
<td>0.052</td>
</tr>
<tr>
<td>Mean absolute error</td>
<td>0.026</td>
</tr>
<tr>
<td>Mean absolute percent error</td>
<td>0.023</td>
</tr>
<tr>
<td>Theil’s inequality coefficient</td>
<td>0.018</td>
</tr>
<tr>
<td>Bias proportion</td>
<td>0.047</td>
</tr>
<tr>
<td>Variance proportion</td>
<td>0.001</td>
</tr>
</tbody>
</table>

**Conclusion**

The volatility in Nigeria’s inflation series can be attributed to several economic factors. Some of these factors are money supply, exchange rate depreciation, petroleum price increase, and poor agricultural production. Box-Jenkins’ Seasonal Autoregressive Integrated Moving Average (SARIMA) was employed to analyze monthly inflation rates in Nigeria. The study mainly intends to forecast the monthly inflation rates and observe its trend and influence on the smooth running of the economy. Series of tentative models were developed to forecast Nigeria’s monthly inflation, but based on minimum AIC, BIC and HQC values, and after the estimation of parameters and series of diagnostic test were performed, \( \text{SARIMA (1.1.1) } \times (0.0.1) \), model was judged to be the best model for forecasting after satisfying all model assumptions. The forecast results revealed an unstable pattern in the Nigeria’s inflation rates, which may serve as an eye opener to economic and monetary policy makers in the country.

**REFERENCES**


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